Small-worlds, complex networks and random graphs

R.W. van der Hofstad[1]

[1] Eindhoven University

Empirical findings have shown that many real-world networks share fascinating features. Indeed, many real-world networks are small worlds, in the sense that typical distances are much smaller than the size of the network. Further, many real-world networks are scale-free, in the sense that there is a high variability in the number of connections of the elements of the networks making them highly inhomogeneous.

Spurred by these empirical findings, many models have been proposed for such networks. In this lecture series, we discuss empirical findings of real-world networks, and describe some of the random graph models proposed for them. In particular, we will discuss the classical Erdös-Rényi random graph, and then move to the more relevant configuration model, generalized random graphs and preferential attachment models. We then discuss the small-world phenomenon in these random graph models and its link to 'six degrees of separation'. We highlight some of the ingredients used in the proofs, namely the tree-like nature of the random graphs under consideration and their connection to branching processes.

We close by discussing the more recent problems of information diffusion on random graphs, where the edges are equipped with general independent and identically distributed edge weights leading to first-passage percolation. Such first-passage percolation problems can also be used to describe epidemics on networks, and competition between different species exploring a network.

A rough outline of the lecture series is as follows:

Lecture 1: Real-world networks and random graphs

Lecture 2: Small-world phenomenon in random graphs

Lecture 3: Shortest-weight routing on weighted random graphs

This lecture series is based on joint work with, amongst others: Gerard Hooghiemstra, Shankar Bhamidi, Júlia Komjáthy, Piet Van Mieghem, Henri van den Esker, Enrico Baroni and Dmitri Znamenski.