Random cliques in random graphs

Oliver Riordan [1]

[1] University of Oxford

We show that for each $r \geq 4$, in a density range extending up to, and slightly beyond, the threshold for a K_r-factor, the copies of K_r in the random graph $G(n,p)$ are randomly distributed, in the (one-sided) sense that the hypergraph that they form contains a copy of a binomial random hypergraph with almost exactly the right density. Thus, an asymptotically sharp bound for the threshold in Shamir's hypergraph matching problem – recently announced by Jeff Kahn – implies a corresponding bound for K_r-factors in $G(n,p)$. The same method works for certain other graphs F in place of K_r; the case $r = 3$ is only partially resolved, surprisingly.