Arithmetic removal lemmas for triangles and k-cycles

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Let p be a fixed prime. A k-cycle in $\mathbb{F}_p^n$ is an ordered k-tuple of points that sum to zero; we also call a 3-cycle a triangle. Let $N = p^n$, (the size of $\mathbb{F}_p^n$). Green proved an arithmetic removal lemma which says that for every k, $\epsilon > 0$ and prime p, there is a $\delta > 0$ such that if we have a collection of k sets in $\mathbb{F}_p^n$, and the number of k-cycles in their cross product is at most a δ fraction of all possible k-cycles in $\mathbb{F}_p^n$, then we can delete ϵN elements from the sets and remove all k-cycles. Green posed the problem of improving the quantitative bounds on the arithmetic triangle removal lemma, and, in particular, asked whether a polynomial bound holds. Despite considerable attention, prior to our work, the best known bound for any k, due to Fox, showed that $1/\delta$ can be taken to be an exponential tower of twos of height logarithmic in $1/\epsilon$ (for a fixed k).

In this talk, we will discuss recent work on Green's problem. For triangles, we prove an essentially tight bound for Green's arithmetic triangle removal lemma in $\mathbb{F}_p^n$, using the recent breakthroughs with the polynomial method. For k-cycles, we also prove a polynomial bound, however, the question of the optimal exponent is still open.

The triangle case is joint work with Jacob Fox, and the k-cycle case with Jacob Fox and Lisa Sauermann.