Counting H-free graphs for bipartite H

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For a graph H, the extremal or Turán number $\mathrm{ex}(n, H)$ is the maximum number of edges in a graph on n vertices containing no copy of H as a subgraph. For any H containing a cycle, it was conjectured by Erdős that the number $f_n(H)$ of H-free graphs on n vertices is $2^{(1+o(1))\mathrm{ex}(n,H)}$. This has long been known to be true for graphs with chromatic number $\chi(H) \geq 3$, but does not hold in general for bipartite H. It is instead conjectured that $f_n(H) = 2^{O(\mathrm{ex}(n,H))}$; to date, this has been shown for relatively few examples, and often with considerable difficulty. I'll discuss some recent joint work with Asaf Ferber and Wojciech Samotij where we prove this conjecture for any bipartite H that satisfies a conjecture of Erdős and Siminovits on the asymptotic behavior of $\mathrm{ex}(n, H)$.