Around Hilbert's 16th Problem
Conférence in honnor of Jean-Jacques Risler

Abstracts

Lev Birbrair : *Metric Geometry of Complex Algebraic Surfaces*

We consider algebraic sets (real or complex) as metric spaces with a natural inner metric, obtained from the embeddings of these sets into the affine (or projective) space. We interest in the cases when the sets have singularities (isolated or nonisolated) and study the properties of these metrics near singular points. In particular, we consider the question of Lipschitz classification of singularities of curves and surfaces. For real surfaces we present a complete answer, but the case of complex surfaces is more difficult...

Felipe Cano : *Birational reduction of singularities of vector fields*

We give a global result of reduction of singularities in the birational context, for vector fields in dimension three. This is obtained by doing a Local Uniformization in the sense of Zariski, followed by a global patching theorem. The Local Unifromization will be valid in any dimension.

Benoît Chevallier : *Maximality and symmetry breaking*

Alexander Degtyarev : *Real elliptic surfaces and real trigonal curves of type I*

We attempt to study/classify real Jacobian elliptic surfaces of type I or, equivalently, separating real trigonal curves in geometrically ruled surfaces. (On the way, we extend the notions of type I and being separating to make them more suitable for elliptic surfaces.) We reduce the problem to a simple graph theoretical question and, as a result, obtain a characterization and complete classification (quasi-simplicity) in the case of rational base. (The results are partially interlaced with those by V. Zvonilov.) As a by-product, we obtain a criterion for a trigonal curve of type I to be isotopic to a maximally inflected one.

Andrei Gabrielov : *Dessins d'Enfants for the Eigenfunctions of the Quartic Oscillator*

Eigenfunctions of the Schrödinger operator on the real line with an even polynomial potential of degree four are associated with properly embedded infinite planar trees. The braid group action on the trees helps to understand the

dependence of the eigenfunctions and the corresponding eigenvalues on the coefficients of the potential.

Danielle Gondard-Cozette : *On the number of connected components of smooth real varieties*

I will present a formula giving the number of connected components of a smooth projective variety over R, and a criterion for their separation. Sketches of proofs will lead us to use facts coming from R-places and higher level orderings in the function field of the variety.

Pedro Gonzales Perez : *Multi-Harnack smoothings of real plane branches*

A smoothing of a germ $(C, 0)$ of real algebraic plane curve singularity is a real analytic family of real algebraic plane curves C_t, for $t \in [0, 1]$, such that $C_0 = C$ and $C_1 = C'$ and C_t for $0 < t \ll 1$ is non singular and transversal to the boundary of a Milnor ball B of the singularity $(C, 0)$. The real part $\mathbf{R}C'$ of C' in the Milnor ball consists of finitely many ovals and non closed components. A M-real algebraic curve (resp. a M-smoothing) reaches the *Harnack bound* on the number of connected components of the real part (in the Milnor ball). For a real plane branch $(C, 0)$ the existence of M-smoothings was shown by Risler by using the blow-up method, a geometrical construction which adapts classical methods of Harnack.

In this talk I will present a joint work with J.-J. Risler. We introduce a new method for the construction of M-smoothings of a real plane branch $(C, 0)$ which are obtained as the result of a *sequence of M-smoothings* of the strict transforms $(\tilde{C}_j, o_j)$ at of C at certain infinitely near points in a embedded resolution of $(C, 0)$ constructed with toric morphisms. These intermediate smoothings are constructed by using Viro method to glue the charts of suitable M-curves in certain real projective toric surfaces determined by the equisingularity class of $(C, 0)$. We analyze the class of *multi-Harnack smoothings*, those smoothings arising in a sequence M-smoothings of the strict transforms $(\tilde{C}_j, o_j)$ which are in *maximal position* with respect to the coordinate axes. Using a result of Mikhalkin in the projective toric case, we prove that their topological type is determined by classical equisingularity class of $(C, 0)$, or equivalently by, the embedded topological type of $(C, 0) \subset (\mathbf{C}^2, 0)$.

Victor Goryunov : *Complex crystallographic groups and symmetries of parabolic functions*

Reflection groups have been traditionally associated with singularities. At first, these were Weyl ADE groups which classified simple functions. After that, the BCF followed in the boundary context and some of the Shephard-Todd in the equivariant We will discuss the first singularity appearance of the complex crystallographic groups, in equivariant monodromy of parabolic functions. The relation between the discriminants, of the functions and of the groups, will be also indicated.

Alcides Lins Neto :

Hossein Movasati : *The infinitesimal Hilbert's 16th problem*

We discuss some problems related to the Hilbert's 16th problem on limit cycles and the role of abelian integrals. Further, we study the analogue of the infinitesimal 16th Hilbert problem in dimension zero. Lower and upper bounds for the number of the zeros of the corresponding Abelian integrals (which are algebraic functions) are found. Finally, we give necessary and sufficient conditions for an Abelian integral to be identically zero.

Daniel Panazzolo : *The group of translations and powers: estimating the number of fixed points, a theorem of S. Cohen and a conjecture of G. Higman.*

The group generated by the real translations $x \mapsto x + a$ and powers $x \mapsto x^r$ appears in several contexts (dynamical systems, 16 Hilbert's Problem and algebraic geometry). How many fixed points can have an element of this group? Is this group a free product?

Bernard Teissier : *Introduction to non archimedian amoebas*

I will present some of the ideas of non archimedian amoebas for hypersurfaces and show how they can be realized as sections of a (non rational) fan.

Oleg Viro : *Strangeness*

Jean Yves Welschinger :

Yosef Yomdin : *Generalized Centre-Focus problem for Abel Equation*

The classical Center-Focus problem, as stated for the Abel differential equation $y' = p(x)y^2 + q(x)y^3$, is to give conditions on the polynomials p and q for $y(a) = y(b)$ for all the solutions $y(x)$. This question leads to some interesting problems in Composition Algebra of polynomials and in Generalized Moments. The Generalized Center-Focus problem is to give conditions under which two Abel equations have the same Poincare mapping on $[a, b]$, and to reconstruct the Abel equations from its Poincare mapping. This generalization leads to certain (presumably important) new questions in Composition Algebra and Moments which may clarify also certain aspects of the original problem. In a somewhat surprising way this problem turns out to be closely related to certain questions in robust Signal Processing.