VIII BRAZILIAN WORKSHOP ON CONTINUOUS OPTIMIZATION

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ABSTRACT LIST

NOTES:

1. Speakers' names are **boldfaced**.
2. Abstracts are shown in alphabetical order of the speakers' family names.
3. Session codes after titles refer to the workshop program.

Dual convergence for penalty proximal point algorithms in convex programming (Mo1)

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We consider an implicit iterative method in convex programming which combines inexact variants of the proximal point algorithm, with parametric penalty functions. We investigate a multiplier sequence which is explicitly computed in terms of the primal sequence generated by the iterative method, providing some conditions on the parameters in order to ensure convergence towards a particular dual optimal solution.

Convex representations of maximal monotone operators in nonreflexive Banach spaces (Th3)

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In this talk we present recent results in collaboration with B.F.Svaiter on convex representations of maximal monotone operators on nonreflexive Banach spaces. The focus will be on the use of the Fitzpatrick and Burachik-Svaiter functions for obtaining results on monotone operator theory (on nonreflexive spaces) of type: surjectivity of perturbations by duality mappings, uniqueness of the extension to the bidual, Brøndsted-Rockafellar property, etc.

On generalizations of the AGP optimality condition (Fr1)

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The Approximate Gradient Projection (AGP) is a practical sequential optimality condition introduced in [Martínez and Svaiter, 2003], used in several important algorithms. In this work we introduce generalizations of the AGP condition with practical and theoretical consequences. Moreover relations are established with the Constant Positive Linear Dependence constraint qualification (CPLD) [Andreani, Martínez and Schuverdt, 2005].

Partial spectral projected gradient method with active-set strategy for linearly constrained optimization (Tu4)

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A method for linearly constrained optimization which modifies and generalizes recent box-constraint optimization algorithms is introduced. The new algorithm is based on a relaxed form of Spectral Projected Gradient iterations. Interlaced with these projected steps, internal iterations restricted to faces of the polytope are performed, which enhance the efficiency of the algorithms. Convergence proofs are given and numerical experiments are included and commented.

Numerical solution of a minimax optimal control problem using the Pontryagin maximum principle (Tu3)

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We consider in the interval $[0,T]$ a dynamic system which evolves according to the ordinary differential equation

$$\left| \begin{array}{ll} \dfrac{dy}{ds}(s) = g(y(s),\alpha(s)) & 0 \le s \le T, \\ y(0) = x \in \Omega \subseteq \mathbb{R}^r, & \Omega \ \texttt{an open domain.} \end{array} \right. \tag{1}$$

The optimal control problem consists in minimizing the functional J

$$J : (x, \alpha(\cdot)) \in \Omega \times \mathcal{U} \mapsto \texttt{ess sup} \{f(y(s), \alpha(s)) : s \in [0, T]\} . \tag{2}$$

The set of controls is

$$\mathcal{U} = \{\alpha : [0, T] \to A \subset \mathbb{R}^m : \alpha(\cdot) \ \texttt{measurable}\}$$

and the set of controls A is compact.

This problem arises, for example, when we want to minimize the maximum deviation of the controlled trajectories with respect to a given special trajectory. This differs from those problems usually considered in the optimal control literature, where an accumulated cost is minimized. As considering an accumulated cost is not always the best method to qualify a controlled system with an unique real parameter, problems of this type has received considerable interest (see e.g. Barron and Ishii (1989), Barron E.N. (1990), and Barles G. et al. (1994)).

The objective of this work is to approximate in a numerical way an optimal control policy $\hat{\alpha}$ such that:

$$J(x,\, \hat{\alpha}(\cdot)) = u(x) := \inf \{J(x, \alpha(\cdot)) : \alpha \in \mathcal{U}\} . \tag{3}$$

Hypothesis. We assume that f, g are bounded and uniformly continuous functions on $\Omega \times A$ and there exists $\partial g/\partial x$ which is a bounded and continuous function also on $\Omega \times A$.

We present here a simplified version of the problem, where we suppose that

$$\left| \begin{array}{l} g(y, \alpha) = g_1(y) + g_2(y)\, \alpha \\ A \text{ is convex} \\ f \text{ is independent of } \alpha. \end{array} \right. \tag{4}$$

We also suppose that the trajectory $y(\cdot)$ remains in Ω, for any control belonging to $\mathcal{U}$.

Results. The principal results obtained in our work are the following ones:

- We obtain a set of necessary conditions that must be verified by any optimal control policy.
- These necessary conditions are given in terms of Lagrange multipliers (adjoint vectors), as it is usual in Pontryagin's approach.
- We define an approximation in discrete time of the original problem.
- The solutions of the approximated problem converge to the continuous solution with a rate h.
- We present a computational algorithm to obtain the solution of the discrete problem and numerical results.

Augmented Lagrangian and splitting proximal algorithms for variational inequalities: applications to dynamical games, PDE's and optimal control (ThP4)

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We review some recent progress on splitting algorithms for variational problems. It is worth seeing the algorithms as discrete time versions of continuous dissipative dynamical systems. We first consider convex variational problems, in which case multipliers and Lagrange duality approach naturally apply. Then, we consider nonconvex nonsmooth "tame" optimization where Kurdyka-Lojasiewicz inequality plays a basic role.

a) Let us consider structured *convex* minimization problems

$$(P) \quad \min\{f(x) + g(y) + \phi(x, y) : \quad Ax - By = 0\}$$

where $\mathcal{X}$, $\mathcal{Y}$ and $\mathcal{Z}$ are real Hilbert spaces, $f : \mathcal{X} \to \mathbb{R} \cup \{+\infty\}$, $g : \mathcal{Y} \to \mathcal{R} \cup \{+\infty\}$ are closed convex proper functions, A, B are linear continuous operators acting respectively from $\mathcal{X}$ and $\mathcal{Y}$ into $\mathcal{Z}$ and $\phi : \mathcal{X} \times \mathcal{Y} \to \mathbb{R}^+$ is a convex quadratic function. Our approach relies on an alternate proximal minimization/maximization procedure applied to the *augmented Lagrangian* function

$$L_\lambda(x, y, z) = f(x) + g(y) + \phi(x, y) + \langle z, Ax - By\rangle + \frac{\lambda}{2} \parallel Ax - By \parallel^2_{\mathcal{Z}} .$$

We study the following splitting algorithm, in which proximal minimization steps are performed alternatively on the primal variables x and y, and then a proximal maximization step is performed on the dual variable z:

$(x_0, y_0, z_0) \in \mathcal{X} \times \mathcal{Y} \times \mathcal{Z}$, $\lambda > 0$ given , $(x_k, y_k, z_k) \to (x_{k+1}, y_{k+1}, z_{k+1})$ as follows

$$\begin{cases} x_{k+1} = \operatorname{argmin}\{f(\xi) + \phi(\xi, y_k) + \langle z_k, A\xi\rangle + i\frac{\lambda}{2} \parallel A\xi - By_k \parallel^2_{\mathcal{Z}} + \frac{1}{2\lambda} \parallel \xi - x_k \parallel^2_{\mathcal{X}} : \ \xi \in \mathcal{X}\} \\ y_{k+1} = \operatorname{argmin}\{g(\eta) + \phi(x_{k+1}, \eta) - \langle z_k, B\eta\rangle + \frac{\lambda}{2} \parallel B\eta - Ax_{k+1} \parallel^2_{\mathcal{Z}} + \frac{1}{2\lambda} \parallel \eta - y_k \parallel^2_{\mathcal{Y}} : \ \eta \in \mathcal{Y}\} \\ z_{k+1} = z_k + \lambda(Ax_{k+1} - By_{k+1}). \end{cases}$$

Just assuming the set of equilibria to be non empty, we show that, for each initial data (x_0, y_0, z_0), the algorithm generates a sequence (x_k, y_k, z_k) which weakly converges to a saddle point of the Lagrangian function $L(x, y) = f(x) + g(y) + \phi(x, y) + \langle z, Ax - By\rangle$. This alternating algorithm, which stems from the seminal work of Glowinski and allied, proves to be flexible and allows many variants. Applications are given in game theory (Nash equilibria), PDE's (domain decomposition) and optimal control (constraint on the control and/or the state).

b) In the *nonconvex* case, primal aspects of the above algorithm have been recently studied in a joint work with J. Bolte, P. Redont and A. Soubeyran. Given a (nonconvex) function $F(x, y) = f(x) + g(y) + \phi(x, y)$ which satisfies Kurdyka-Lojasiewicz inequality, it is proved that the sequence generated by the alternating proximal minimizing algorithm has a finite length, and hence converges to a critical point of F. The algorithm can be viewed as a proximal regularization of the usual Gauss-Seidel method. As a striking application, one obtains the convergence of our alternating projection algorithm (a variant of Von Neumann algorithm) for a wide class of sets including semialgebric and tame sets, transverse smooth manifolds or sets with regular intersection.

A new method for nonlinear programming with extension to variational inequalities (MoP4)

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For nonlinear programming all the existing methods (interior, exterior, penalty, feasible methods) are based on a subproblem to be solved, and the efficiency of the whole method depends highly on the nature and the structure of this subproblem. In this talk we present a new method where at each step the subproblem has a very nice structure, and can be solved very efficiently via duality theory. Convergence to stationary points is proved under mild assumptions. In the convex case, if in addition the objective function is strongly convex, then it is proved that the whole sequence converges linearly to the optimal solution. Similar results are proved for the extension of the method for solving variational inequalities. The algorithm is particularly adequate for large scale problems, and numerical results will demonstrate the viability and efficiency of the algorithm when compared to some state-of-the-art methods/software, such as SQP and others.

An explicit method for variational inequalities (Mo2)

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We introduce a fully explicit method for solving monotone variational inequalities in Hilbert spaces, where orthogonal projections onto the feasible set are replaced by projections onto suitable hyperplanes. We prove weak convergence of the whole generated sequence to a solution of the problem, under the only assumptions of continuity and monotonicity of the operator and existence of solutions.

Outer Trust-Region method for constrained optimization (Th2)

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Nonlinear programming algorithms are iterative. Usually, iterations include acceptance criteria that ensure global convergence to reasonable points, independently of the initial approximations. However, not all the possible limit points are of the same quality. Sometimes limit points are not even feasible or are local minimizers with poor objective function value. This state of facts motivates additional acceptance criteria for NLP iterates. In this paper we consider the ALGENCAN implementation of the PHR Augmented Lagrangian algorithm and we introduce an outer trust region scheme, by means of which an iterate is not accepted if infeasibility increases too much. It is proved that usual global convergence theorems under the CPLD constraint qualification also hold for the new algorithm. Extensive numerical experimentation is presented.

A primal-dual modified subgradient algorithm with Augmented Lagrangians (TuP2)

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We apply a modified subgradient algorithm (MSG) for solving the dual of a nonlinear and nonconvex optimization problem. The dual scheme we consider includes the sharp augmented Lagrangian as a particular case. An attractive feature of our method is primal convergence, which means that every accumulation point of a primal sequence (which is automatically generated by the algorithm), is a primal solution. Primal convergence cannot be achieved in general by the current variants of MSG available in the literature. We propose two new variants of MSG which enjoy both primal and dual convergence, as long as the dual optimal set is nonempty. These variants involve stepsizes which are very simple to calculate and implement. We establish primal convergence even in the case when the dual optimal set is empty. Finally, we show that our second variant of MSG converges in a finite number of steps.

A generalized like-distance in convex programming (Fr3)

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We consider a generalized distance-like which contains as degenerate cases ϕ-divergences and distance-likes with second order homogeneous kernels. The motivation to get this distance-like comes from the study of shifted penalty function in the primal space. The conjugate of these penalty functions need not pass through the origin with slope one, and for a particular case we get a generalization of the Kullback-Liebler entropy distance. The new distance-like can be seen as the difference between a Bregman function and its linear approximation at specific values of the arguments. We show dual and primal convergence results: each limit point of the sequence generated by the proximal method defined by the generalized distance-like applied to the dual problem is an optimal dual solution.

Alfredo, from Feasibility to Optimization and Inverse Problems (MoP2)

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We remember work with Alfredo in feasibilty and optimization and its relationship with very recent methodologies for solving Inverse Problems, especially in Image Reconstruction.

Derivative-free optimization (Th4)

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Since in many industrial and scientific applications, the derivatives of the functions cannot be computed, as, for instance, when the objective function is evaluated through a computer simulation process, the interest in studying derivative–free optimization methods is increasing very fast. Our research group has been working with this kind of methods for unconstrained minimization problems and, more recently, for solving the general problem of optimization, with equality and inequality constraints. In this work, we intend to summarize our works concerning the unconstrained problems and also to describe the proposed approach for constrained minimization that consists of a derivative-free Augmented Lagrangian method. It is based on the algorithm introduced by Andreani, Birgin, Martínez and Schuverdt, in 2007, with the calculations of derivatives replaced by suitable modifications. It was proved that all the important theoretical results of the original method, that uses the calculus of the derivatives, were kept in our derivative-free scheme. We will also present recent numerical experiments obtained for solving a problem of mechanic engineering.

The Constant Rank Condition is a second order constraint qualification (Fr1)

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The Constant Rank condition for feasible points of nonlinear programming problems was defined by Janin. In that paper the author proved that the condition was a first order constraint qualification. In this work we establish that the Janin Constant Rank condition is, in addition, a second order constraint qualification. We define other second order constraint qualifications.

Proximal methods for nonlinear programming: double regularization and inexact subproblems (FrP1)

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This talk presents a computational study of techniques that might be incorporated into large-scale, parallelizable augmented Lagrangian methods for nonlinear optimization. We solve the subproblems by a modern conjugate-gradient-based boxed-constrained first-order method due to Hager and Zhang. In this context, we explore the usefulness of various recent theoretical developments in proximal methods, such as double-regularized coercive distance kernels and relative error criteria for early termination of subproblems. We also compare primal-dual and pure dual regularization approaches. The results are different than those obtained for complementarity problems in our earlier work, but broadly agree in several respects with a recent study by Birgin, Castillo, and Martínez. In conclusion, we discuss where best to next focus our theoretical and computational efforts in this area.

Continuous optimization in improving productivity and capacity utilization in manufacturing of seasonal products (Fr4)

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In this paper, yield management is applied to deal with seasonal demand of products and help improve the utilization of manufacturing capacities.

Rate classes for different customers are defined based on the time of ordering and the conditions of payment. A nonlinear programming (NLP) model is developed and optimized to jointly determine the optimum incentives and price discount levels for each rate class. The model aims at maximizing net revenues. It includes nonlinear relationships representing the impact of incentives on capacity utilization as well as the effect of price discounts on customer demand in each season or market segment. The NLP model is of generic structure which would make it applicable to all multi-products manufacturing facilities. The model is applied to determine the optimum incentive and price discount levels for perishable products in a multi-product plant. Computational Results demonstrated that the model is useful in maximizing net revenues and improving capacity utilization of manufacturing facilities.

Allocation of firm-energy rights among hydro agents (Tu2)

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The firm energy is defined as the maximum constant production that can be achieved by a set of generation plants. In case of hydro plants, it was observed that the overall firm energy was greater than the sum of the firm energies obtained separately for each hydro plant, i.e., there is a "synergy" gain whenever a cooperative operation occurs. The existence of "synergic" benefits in the firm energy production immediately leads to the question of their allocation among the hydro plants. This allocation has great commercial importance in the Brazilian system, because the firm energy right of each plant defines its contracting limit, which in turn has a direct effect on the plant's revenues. On the other side, the increase of the constant energy production means that a greater demand could be continuously supplied. Therefore, the key question is to find a distribution of the benefits shares among the hydro plants which is "fair" and stable in the sense that all of them have an incentive to cooperate in the whole group of plants, a so-called core element. In other words, an allocation is fair if all participants receive more benefits by joining the "grand coalition" than by forming sub-coalitions of their own.

The core of a cooperative game formalizes the concept of "fairness" in the allocation of costs and benefits. The problem of computing the core can be formulated as a linear programming problem, where the number of constraints increases exponentially with the number of players. The major difficulty in the calculation of the core in realistic situations is, therefore, the number of constraints. This report discusses the calculation of the core of a cooperative game. In this sense, this work proposes two approaches to handle this difficulty: (i) a MIP algorithm, based on constraint generation, that computes such a core element for a realistic number of players; (ii) a "randomized" procedure, in which a constraint sampling is applied to the core constraints and it can be shown that the resulting randomized solution fails to satisfy only a small portion of the original constraints, provided that a sufficient number of samples is drawn. The key result is to provide an efficient and explicit bound on the measure (probability or volume) of the original constraints that are possibly violated by the randomized solution. This volume rapidly decreases to zero as N is increased, where N is a finite set of constraints that are taken into account. The results of approach (i) can be used to check optimality of approach (ii).

A quasi-Newton strategy for the stabilized sequential quadratic programming method for variational inequality and optimization problems. (Th2)

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The quasi-Newton strategy presented in this work preserves one of the most important features of the stabilized sequential quadratic programming (sSQP) method, the local convergence without constraint qualifications assumptions. Quadratic convergence of the primal-dual sequence of the sSQP method had been established assuming only the second-order sufficient condition. In this work we show that, when the matrices of the quasi-Newton strategy are updated in the classical form, the generated sequence of matrices is bounded and the primal-dual sequence converges to a solution with superlinear rate.

A reliable method for obtaining the complete weakly efficient set of nonlinear biobjective optimization problems (Th1)

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Multiobjective optimization problems are ubiquitous. Many real-life problems require taking several conflicting points of view into account. Ideally, solving a multiobjective problem means obtaining the whole efficient set, that is, all the points which are efficient (not dominated). That set might be described analytically as a closed formula, numerically as a set of points, or in mixed form as a parameterized set of points. Unfortunately, for the majority of multiobjective optimisation problems, it is not easy to obtain such a description, since the efficient set includes typically a very large number or infinite number of points. The methods proposed in the literature with that purpose are specialized either for particular problems or for a particular class of multiobjective problems (for instance, the multiobjective simplex methods for the linear case). The reason for this lack of methods is that even obtaining a single efficient point of a nonlinear multiobjective problem can be a difficult task. That is why some authors have proposed to present to the decision-maker a "representative set" of efficient points which suitably represent the whole efficient set (either by modifying the definition of efficiency or by selecting a finite set of efficient points with the criteria of coverage, uniformity, and cardinality as quality measures) or an "approximation" of the efficient set by means of sets with a simpler structure.

However, a recent interval branch-and-bound method has been proposed in the literature which is able to obtain a superset containing the complete weakly efficient set of (nearly) any nonlinear biobjective optimization problem, up to a pre-specified accuracy. In the talk we will present the method, and will discuss its applicability to general nonlinear biobjective problems. By drawing in the image space that superset the decision-maker can easily see the trade-off between the two objectives, i.e., how one objective improves as the other gets worse. Something similar can be done in the decision space, by drawing the corresponding superset in a color scale depending on the objective value of one of the objectives.

The interval B&B method deals with the multiple objectives directly. It starts with an initial box containing the feasible set. The box considered is either sent to the solution list, it is removed from further consideration by a "discarding test", or it is split into several subboxes which are considered later. This process is repeated by choosing a new box until no box remains to be considered.

We also give theoretical results which show the worth of the approach. The limit properties, when the tolerances are set equal to zero and the algorithm does not stop, prove that, under mild conditions, the algorithm converges to the set of weakly efficient points. And in the more practical case that the tolerances used are strictly positive and then the algorithm stops after a finite number of iterations, it is proved that the points in the boxes of the solution list are very close to be efficient.

Notice that the use of interval tools make the method very general, in the sense that it can be applied to nearly any nonlinear biobjective optimization problem. Thus, its applicability to many and varied problems seems to make it of potential interest for the scientific community.

In particular, here we have applied the method to a competitive facility location problem. Competitive location deals with the problem of locating facilities to provide a service (or goods) to the customers (or consumers) of a given geographical area where other competing facilities offering the same service are already present (or will enter to the market in the near future). Many competitive location models are available in the literature. However, the literature on multiobjective competitive location models is rather scarce. This is in part due to the fact that single objective competitive location problems are difficult to solve, and considering more than one objective makes the problem near intractable. We study the case of a franchise which wants to enlarge its presence in a given geographical region by opening one new facility. Both the franchisor (the owner of the franchise) and the franchisee (the actual owner of the new facility to be opened) have the same objective: maximize their own profit. However, the maximization of the profit obtained by the franchisor is in conflict with the maximization of the profit obtained by the franchisee. This suggests to use a biobjective model to obtain the efficient solutions for this problem, so that later on the franchisor and the franchisee can agree in both location and design for the new facility, taking the corresponding economical implications of their selection into account. The computational results show the applicability of the method.

Local convergence of the proximal point method for a special class of nonconvex functions on Hadamard manifolds (Tu1)

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Local convergence analysis of the proximal point method for special class of nonconvex function on Hadamard manifold is presented in this paper. The good definition of the sequence generated by the proximal point method is guaranteed. Moreover, it is proved that each cluster point of this sequence satisfies the necessary optimality conditions and, under additional assumptions, its convergence to a minimizer is obtained.

Computational optimization of wireless telecommunication networks (Fr4)

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More and more in a global economy, businesses require access to high speed voice, data and video communications. A region that achieves broadband technology assures itself a place on the high side of the digital divide that separates areas of economic vitality from those left behind. Thus, there is a strong demand for the communication industry on delivering broadband access

for all. Wireless mesh networks are ranked as promising technique for broadband coverage and capacity extension. The data between nodes constructing a wireless mesh network is transmitted on wireless links by means of wireless transmission technologies. These wireless links suffer from time-variant influences caused by the timely changing environment and, as a result, the capacity (maximum amount of data that can be transmitted) of a wireless link is not fixed. Instead, this capacity has to be adjusted by resource allocation. As a consequence, efficiently transmitting data in wireless mesh networks requires integrated routing and resource optimization.

A mathematical analysis of the underlying model leads to a method that solves the corresponding joint routing and power control optimization problem by decomposing the problem into sub-problems while still meeting main requirements such as distributed implementation, multiple path routing, and per-hop error performance. Scheduling is managed separately by including the corresponding requirements in the constraints of the problem. We show that the resulting Routing and Power Control Decomposition (RPCD) algorithm produces a sequence of points converging to a KKT-point of the optimization problem. This convergence holds even if the links suffer multiple access interference, as it is the case for TDMA/CDMA networks. Numerical results show the applicability of the algorithm in practice.

A Gauss-Newton-type method for differentiable exact penalties (Mo4)

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During the 1970's and 1980's, methods based on differentiable exact penalty functions were developed to solve constrained optimization problems. The main drawback of these functions is that they contain second-order terms in their gradient's formula, which did not allow the use of Newton-type methods. With this in mind, we propose a Gauss-Newton-type method using the exact penalty for variational inequality problems, introduced recently by André and Silva. We also extend their penalty function to both equality and inequality constraints using a weaker assumption, and prove convergence results. We conclude with some numerical experiments.

Single-directional property of multivalued maps and lack of metric (Th3)

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Dontchev and Hager have shown that a monotone set-valued map defined from a Banach space to its dual which satisfies the Lipschitz-like property around a point (x, y) of its graph is actually single-valued in a neighbourhood of x. We prove a result which is the counterpart of the above for quasimonotone set-valued maps, based on the concept of single-directional property. As applications, we provide a result on the non-metric regularity of the solution map of general variational systems and of quasivariational inequalities. We also investigate the single-directionality property for the normal operator to the sublevel sets of a quasiconvex function.

A proximal-like algorithm with regularized logarithmic barrier (Mo1)

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We are concerned with the convex optimization problem $\min\{f(x) \mid x \in \mathbb{R}^p_+\}$ where $f : \mathbb{R}^p \to \mathbb{R} \cup \{+\infty\}$ is a closed proper convex function. To solve this problem we consider the following proximal-like scheme: starting from any $x^0 \in \mathbb{R}^p_{++}$ generate $\{x^k\}$ by

$$x^k = \arg\min_{x\in\mathbb{R}^p} \left[f(x) + \lambda_k^{-1} d(x, x^{k-1}) \right],$$

where $\{\lambda_k\}_k \subset \mathbb{R}_{++}$ is an exogenous sequence of parameters and $d : \mathbb{R}^{2p}_{++} \to \mathbb{R}$ is given by

$$d(x,y) = \frac{\nu}{2}||x-y||^2 + \mu \sum_{j=1}^{p} \varphi(y_j^{-1}x_j) \quad \texttt{and} \quad \varphi(r) = r - ln(r) - 1. \tag{5}$$

Our specific choice of the regularized logarithmic barrier functional in (5) relies in results found in papers by Auslender, Teboulle and Ben Tiba, and by Attouch and Teboulle, where the authors studied this proximal scheme with

$$d(x,y) = \frac{\nu}{2}||x-y||^2 + \mu \sum_{j=1}^{p} y_j^2 \varphi(y_j^{-1}x_j) \tag{6}$$

and

$$d(x,y) = \frac{\nu}{2}||x-y||^2 + \mu \sum_{j=1}^{p} y_j \varphi(y_j^{-1}x_j) \tag{7}$$

respectively. Observe that from the numerical point of view and in order to solve the proximal subproblems it is needed to solve the equation for finding o zero of the subdifferential of $f(x) + \lambda_k^{-1} d(x, x^{k-1})$, thus the second derivative of $d(\cdot, x^{k-1})$ has a significant effect. Note that the second derivatives are

$$d''_x(x,y) = y^2 \frac{\mu}{x^2} \quad \texttt{and} \quad d''_x(x,y) = y \frac{\mu}{x^2}$$

respectively. Since from the numerical point of view and in place of y we put the previous iterate (which will be close to the next one in a prox scheme), the trajectory that gets close to the boundary of $\mathbb{R}^p_+$ (thus with $1/x$ big) has the effect of being forced to stay inside diminished by the value of y (which will be close to x, thus to zero) and the "numerical" trajectory could crash to the boundary without attaining "convergence".

With our choice of d we get $d''_x(x,y) = \frac{\mu}{x^2}$. Thus, we hope we can avoid the undesired "numerical" behavior discussed above.

Using different gradient formulae for inverse problems in ODE models (Fr3)

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Two main formulae used to compute gradients in the numerical solution of inverse problems in ODE models are extended. The extension takes into account the dependence of the calculated step length upon the parameter vector. Numerical experiments compare the behavior of all these formulae in solving inverse problems for stiff ODE models.

Nesterov's optimal descent method for differentiable convex functions and extensions (TuP3)

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Descent methods based on first order models play a central role in continuous optimization, and are very convenient for large scale problems. Computational complexity theory establishes upper bounds for the efficiency of algorithms for minimizing convex functions defined in n-dimensional Euclidean spaces. Nesterov's works show that the classical Cauchy steepest descent method cannot reach the best possible complexity bound, and propose new algorithms that do achieve it. The complexity bounds depend on a Lipschitz constant for the first derivatives, and Nesterov's approach depends on this constant, which must be either known or adaptively estimated.

In this talk we describe extensions to Nesterov's approach which do not depend on any Lipschitz constant, and keep the optimal complexity whenever such constant exists. We show a simplified complexity analysis of the algorithms and computational tests.

A projected gradient method for vector optimization with relative error and inexact line searches (Th1)

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We propose an inexact projected gradient-like method for general constrained vector optimization problems. The method admits relative errors on the search directions and the stepsizes are chosen using an Armijo-like rule, implemented with a backtracking procedure. Under some reasonable hypotheses, global convergence to weakly efficient points of all sequences produced by the method is established for convex (respect to the ordering cone) objective functions.

Constrained optimization by low rank updating (TuP1)

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We consider SQP type algorithms for nonlinear programming that avoid any derivative matrix evaluations. In other words not only the Hessian of the Lagrangian but also the Jacobian of the constraints is approximated by low rank updating. Storage and linear algebra operations can be reduced by null- or range space implementations with full or compact representations of low rank corrections. In particular we report numerical results on a limited memory version for large scale problems. arising in optimal applications.

Risk-averse adaptive strategies for large-scale multistage stochastic linear programming (Tu2)

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We consider risk-averse formulations of large-scale multistage stochastic linear programs having a structure that often appears in real-life applications. Specifically, the optimization problem corresponds to controlling a system whose dynamics is given by a transition equation depending affinely on an interstage dependent stochastic process. We device a sliding strategy that for each time step defines a risk-averse problem with constraints that are deterministic for the current time step and uncertain for future times. To each uncertain constraint corresponds both a chance and a Conditional Value-at-Risk constraint. We show that the resulting risk-averse problems are numerically tractable, being at worst conic quadratic programs. For the particular case when uncertainty appears only in the right-hand side of the constraints, such risk-averse problems are deterministic multistage linear programs. We show how to cast each one of such problems into a dynamic programming scheme and give a detailed dual dynamic programming solution approach. We also give an error bound when approximating the sliding strategy with an alternative that does not use the most recent information but is lighter in terms of computational bulk.

Convex analysis for set and vector valued functions (Fr1)

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New concepts of Fenchel conjugates, subdifferentials and directional derivatives for set–valued convex functions are introduced. Set-valued counterparts to well-known scalar formulas are established: Fenchel-Moreau's theorem, duality theorems of Fenchel-Rockafellar type and max-formulas.

The approach is based on a consequent use of set relations being canonical extensions of the order of an ordered vector space to its power set. This leads for instance to a new set-valued "order difference" making possible a coherent definition of the directional derivative. Moreover, instead of linear operators as in most previous approaches, pairs of linear functionals are used as dual variables.

On critical Lagrange multipliers (ThP1)

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For constrained optimization problems with nonunique Lagrange multiplier associated to a solution, we define the thin subclass of multipliers, called critical (and in particular, violating the second-order sufficient condition for optimality), and possessing some very special properties. Specifically, convergence to a critical multiplier appears to be a typical scenario of dual behaviour of primal-dual Newton-type methods when critical multipliers do exist. Moreover, along with the possible absence of dual convergence, attraction to critical multipliers is precisely the reason for slow primal convergence. On the other hand, critical multipliers turn out to have some special analytical stability properties: noncritical multipliers should not be expected to be stable subject to parametric perturbations of optimality systems.

Algebraic connectivity for classes of caterpillars (Fr2)

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A caterpillar is a tree with at least three vertices for which the removal of all pendant vertices results in a path P_s, $s \geq 2$.

Some ordering of trees by algebraic connectivity with restrictions are known in the literature. The first paper about it appeared in 1990 and it is due to Grone and Merris. They gave a partial order in the class of trees of order $n = k + l$ obtained joining an edge between the centers of two stars S_k and S_l. In 1998, Fallat and Kirkland exhibited the unique (up to isomorphism) tree of order n with specified diameter d that minimize the algebraic connectivity over all such trees. They determine that the tree of order $n = k + l + d - 1$ obtained from a path P_{d-1} and two stars S_{k+1} and S_{l+1} by identifying every pendant vertex of the path with the center of a star has minimum algebraic connectivity among the trees of order n and diameter d. Zhang discusses about how to find a total ordering of trees with diameter 4 and he determines orderings of trees by algebraic connectivity in non-increasing order. They consider the tree of order n obtained from a star of order S_{k+1} and $k \geq 2$ stars S_{p_i+1} of order $p_i + 1, 1 \leq i \leq k$, $p_1 \geq p_2 > 0$ by identifying every pendant vertex of S_{k+1} with the center of a $S_{p_i+1}, 1 \leq i \leq k$. Recently, some other results about ordering of classes of trees by algebraic connectivity were obtained.

We observe that several of the trees that minimize the algebraic connectivity under restrictions are caterpillars. This work presents a survey of these results and study some special cases of caterpillars.

A trust region algorithm for optimization without derivatives (Th2)

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In this work we discuss a trust-region method for box-constrained derivative-free optimization. The method uses quadratic models which are based only on the objective function values. The models are constructed from interpolation conditions without any derivatives as suggested by Powell in his recent algorithm named BOBYQA.

Optimization over the efficient set of multi-objective convex optimal control problems (Tu3)

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We consider multi-objective convex optimal control problems. First we state a relationship between the (weakly or properly) efficient set of the multi-objective problem and the solution of the problem scalarized via a convex combination of objectives through a vector of parameters (or weights). Then we establish that (i) the solution of the scalarized (parametric) problem for any given parameter vector is unique and (weakly or properly) efficient and (ii) for each solution in the (weakly or properly) efficient set, there exists at least one corresponding parameter vector for the scalarized problem yielding the same solution. Therefore the set of all parametric solutions (obtained by solving the scalarized problem) is equal to the efficient set. Next we consider an additional objective over the efficient set. Based on the main result, the new objective can instead be considered over the (parametric) solution set of the scalarized problem. We construct numerical methods, after pointing to existing solution differentiability results for parametric optimal control problems. We give an example application to illustrate our proposed approach.

Optimal auction with a general distribution: virtual valuation without a density (Fr4)

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We characterize the optimal auction in an independent private values framework for a completely general distribution of valuations. We do this by introducing a new concept: the generalized virtual valuation. In order to show the wider applicability of this concept, we present two examples showing

how to extend the classical models of Mussa and Rosen and of Baron and Myerson for arbitrary distributions.

Semismooth methods for a hemivariational inequality (Mo2)

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The main topic is focused on nonsmooth methods of continuous as well as discrete optimization. As the reference model we consider a hemivariational inequality arising in the obstacle problem due to cohesion forces.

From a point of view of continuous optimization, the model of contact with cohesion is described by constrained minimization of the objective functional, which is nonconvex and nondifferentiable. As a consequence, necessary and sufficient optimality conditions for the minimization problem do not coincide. The necessary optimality condition is represented as a hemivariational inequality. However, the operator of the problem in the pure primal formulation does not fit any class of monotone operators. Henceforth, to treat a (non-unique) solution of the problem, we rely on sufficient optimality conditions expressed within a primal-dual formulation. The associated saddle point problem suggests to treat the primal state variable and the dual state variables independently. To provide well-posedness of the primal-dual formulation requires suitable regularization of the nondifferentiable term.

In the framework of numerical optimization, primal-dual active set methods were developed recently to compute efficiently solutions of convex minimization problems. The common advantage lies in the fact that they are associated to generalized Newton methods. There is our first successful attempt to treat the nonconvex minimization problem within this strategy. We construct both: a primal-dual active set algorithm to find a solution of the reference problem, and a generalized Newton algorithm for the regularized problem. Its global convergence properties are analyzed in continuous and finite-dimensional spaces. Based on the maximum principle the monotone convergence properties are established in the continuous setting of the problem, while justification of the global convergence requires discretization of the problem. Further, we incorporate this strategy into an adaptive finite element framework. We will present our numerical findings.

Subdifferential estimate of the classical directional derivative for lower semicontinuous functions (Mo3)

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Let X be a Banach space and $f : X \to \mathbb{R} \cup \{+\infty\}$ be a function. The classical directional derivative of f at point $\bar{x} \in \operatorname{dom} f$ in direction $d \in X$ is defined by

$$f'(\bar{x}; d) := \liminf_{t \searrow 0} \frac{f(\bar{x} + td) - f(\bar{x})}{t} \in \overline{\mathbb{R}}.$$

For a subdifferential ∂ on the class of lower i semicontinuous functions on X, we discuss the following property:

Let $f : X \to \mathbb{R} \cup \{+\infty\}$ be lower semicontinuous. For every $d \in X$ and $\bar{x} \in dom\ f$, there exists a sequence $(\bar{x}_n, \bar{x}_n^*)_n \subset \partial f$ such that $x_n \to_f \bar{x}$ and

(i) $f'(\bar{x}; d) \leq \liminf_{n\to\infty} \langle x_n^*, d\rangle$;

(ii) $\liminf_{n\to\infty} \langle x_n^*, x - x_n\rangle$.

We show that this property is equivalent to weak first-order rules, to Zagrodny's mean value inequality and (for bornological subdifferentials) to the nonemptyness of the subdifferential on a dense subset.

The bounded cycle cover problem (Fr2)

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We will review exact and heuristics algorithms for the Bounded Cycle Cover Problem. BCCP requires to determine a minimum cost cycle cover of a graph, with cycles bounded in length and number of edges. This problem arises in the design of fiber-optic telecommunications networks that employ multiple self-healing rings to provide routing for communication traffic, even in the event of a fiber cut or other kinds of failures. Results of algorithms we developed for the BCCP will be presented.

Outer approximation schemes for generalized semi-infinite variational inequality problems (Mo2)

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We introduce and analyze outer approximation schemes for solving variational inequality problems in which the constraint set is as in generalized semi-infinite programming. We call these problems Generalized Semi-Infinite Variational Inequality Problems (GSIVP). First, we establish convergence results of our method under standard boundedness assumptions. Second, we use suitable Tikhonov-like regularizations for establishing convergence in the unbounded case.

Attitude control problems by Sequential Quadratic Programming (Th2)

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In this work we provide a new approach to the attitude control problem, using a nonlinear programming formulation which has some differential equations among its constraints. We consider a space mission with the task of collecting images using a camera coupled to a mechanical arm. In order to do this task, the mechanical arm is often required to change its orientation (attitude). This must be done in some optimal manner, while ensuring that this sensitive optical instrument is not exposed to very bright objects in the sky, such as the sun. The problems that we study here take into account the orbit of the spacecraft in the space mission, too.

Thus, the purpose is to determine the trajectory of the spacecraft mechanical arm (described by quaternions), from an initial position to the final position, taking into account the orbital law of motion of the spacecraft and the kinematics laws of motion of the mechanical arm. Our main objective is to determine the necessary control torques such that the constraints mentioned before are satisfied.

The problem is formulated as a time optimization problem. The discretized formulation of the optimal control problem is solved by an optimization process, which makes use of sequential quadratic programming.

Some numerical simulations are also presented.

Impact of the crude pre-heating trains in the optimization of a crude distillation unit system (Tu2)

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Industrial Crude Oil Distillation Units (CDU) have relatively small operational windows due to the limitations set by restrictions like hydraulic capacities to pump crude oil and draw-off products, products specifications, hydraulic and separation capacities of the towers and thermal integration with crude oil preheat exchangers train

Techniques like rigorous modeling, linear programming and unit specific empirical correlations have been reported to optimize CDU (Basak 2002; Inamdar 2004; Chau-Kuang 2004). In all these papers, CDU operation optimization deals only with the atmospheric tower and doesn't take into account the thermal integration with preheat exchangers train. Despite this simplification accelerates model development and the model is easy to use, it can produce misleading conclusions and even operational conditions not allowed in the industrial unit.

In this work two optimization models were built of a system comprisingthree atmospheric distillation towers, two vacuum distillation towers and two preheat exchanger trains. The first model only encompasses the optimization of the atmospheric distillation towers and didn't take into account the energetic integration. The later model took into account the additional restrictions imposed by the heat integration. Both models took into account the restrictions set by hydraulic capacities to pump crude oil and draw-off products, products specifications, hydraulic and separation capacities of the towers and maximum, temperatures and heat flow rates of pumparounds and furnaces outlet temperatures.

To develop these integrated models, rigorous models of each tower were initially built in PRO/II. The metamodeling technique was used to develop simplified models from each rigorous PRO/II model using the Latin Hypercube sampling (LHS) technique. To model the preheat exchanger trains mass and energy balances and the design equations of each heat exchanger were programmed. The integrated models were written in GAMS and the optimizations were solved using CONOPT, a non linear programming tools of GAMS.

It was found that when heat integration is not taken into account crude oil throughput and product yields are higher. The temperatures and heat flow rates limits of the pumparounds decreases the separation capacity of the atmospheric towerswhen heat integration is taken into account. Furthermore the operational conditions of the heat integration model are closer to actual conditions and easier to implement.

These results show that despite models encompassing heat integration are longer to develop and more difficult to run they produce better results that are easier to implement in the industrial plant.

Conjugate directions in multicriteria optimization (Th1)

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We present a adapted version of the conjugate directions method for the multicriteria optimization. Our algorithm has finite termination in the case when all objective function are quadratic ones. At each iteration, it requires the computation of the descent direction, conjugate in some sense, which requires the solution of a linear systems, whose dimension increases as the iterations grows. This is the disadvantage of our approach. We discuss also how to formulate the conjugate gradient method for more general cases.

A globalized Newton's method for minimizing matrix functions (Mo4)

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The minimization of unconstrained nonlinear matrix functions via a trust region algorithm is considered. The solution of the trust region subproblem involves the minimization of a quadratic matrix function. A Newton-like method is applied. The Newton step results as the solution of the Riccati matrix equation which can be solved by using a technique based on the sign of a matrix or by the Newton method for nonlinear matrix system. In the last case the step is obtained as the solution of a Sylvester matrix equation.

From the theoretical point of view the existence of the trust region step requires concepts from the vector optimization area. The relationship between the partial order of Loewener in the subspace of symmetric matrices of $\mathbb{R}^n$ and the Pareto optimality of a functional defined on a subset of the Euclidean space $\mathbb{R}^{n^2}$ through the *vec* transformation is discussed, and the existence of the trust region step is proved. Convergence results and preliminary numerical experiments are reported.

Inexact restoration for electronic structure calculations (MoP3)

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Several electronic calculation problems motivate the problem of minimizing a matricial function on the set of symmetric idempotent matrices with a given rank. The number of variables and constraints may be very large. Usual methods for solving this problem involve eigenvalue computations, trust region feasible strategies and direct iterative subspace acceleration. Here we introduce an inexact restoration procedure for solving the problem. The method does not need computation of eigenvalues and may preserve sparsity patterns. Global convergence holds under usual assumptions and, under more restrictive hyptheses, the convergence is quadratic. Extensive numerical experimentation will be shown.

Motzkin decomposition of closed convex sets (Mo3)

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Theodore Motzkin proved, in 1936, that any polyhedral convex set can be expressed as the (Minkowski) sum of a polytope and a polyhedral convex cone. This joint paper with M. A. Goberna, E. Gonzlez and M. I. Todorov provides several characterizations of the larger class of closed convex sets in finite dimensional Euclidean spaces which are the sum of a compact convex set with a closed convex cone. These characterizations involve different types of representations of closed convex sets as the support functions, dual cones and linear systems whose relationships are also analyzed in the paper. The obtaining of information about a given closed convex set F and the parametric linear optimization problem with feasible set F from each of its different representations, including the Motzkin decomposition, is also discussed.

A topological Mountain Pass Lemma and its implications regarding the uniqueness of constrained minimizers (Fr1)

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We present topological and metric versions of the well know Mountain Pass Lemma and explain how they can be used to prove the uniqueness of local minima in nonlinear programs. We consider both finite diminensional and infinite dimensional nonlinear programs subject to the Mangasarian Fromovitz constraint qualification.

Some completion problems (Fr2)

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In 1975 G. N. de Oliveira has proposed the following matrix completion problems:
Let F be a field. Let n, p, q be positive integers such that $n = p + q$. Let

$$A = \begin{bmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \end{bmatrix} \tag{8}$$

be a partitioned matrix, where $A_{1,1} \in F^{p\times p}$, $A_{2,2} \in F^{q\times q}$. Suppose that some of the blocks $A_{i,j}$, $i, j \in \{1, 2\}$ are fixed and the others vary. Under what conditions there exist a matrix of the form (8) with prescribed eigenvalues, characteristic polynomial, invariant factors?

We present the most significant contributions available in the literature concerning this problems.

On the complexity of the hybrid proximal extragradient method for the iterates and the ergodic mean (FrP2)

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We analyze the iteration-complexity of the hybrid proximal extragradient (HPE) method for finding a zero of a maximal monotone operator recently proposed by Solodov and Svaiter. One of the key points of our analysis is the use of a new termination criteria based on the ε-enlargement of a maximal monotone operator. We show that Korpelevich's extragradient method for solving monotone variational inequalities falls in the framework of the HPE method. As a consequence, we obtain new complexity bounds for Korpelevich's extragradient method which do not require the feasible set to be bounded, as assumed in a recent paper by Nemirovski. We also study the complexity of a variant of a Newton-type extragradient algorithm proposed by Solodov and Svaiter for finding a zero of a smooth monotone function with Lipschitz continuous Jacobian.

Korpelevich's method for variational inequality problem in Banach spaces (Mo2)

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We propose a variant of Korpelevich's method for solving variational inequality problems with operators in Banach spaces. We establish the weak convergence of the sequence generated by the method under reasonable assumptions on the problem data.

Maximality and continuity (Th3)

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Maximality is an important concept for monotone and cyclically monotone maps. In this lecture we show that maximality is nothing else that a form of continuity both on the domains of the map and its inverse, and on the maps. Necessary and sufficient conditions for maximality are revisited thanks to this point of view.

In the second part we show that some but not all of these relationships between maximality and continuity hold also for (cyclically) psedomonotone maps.

Bregman proximal algorithms for solving variational inequality problems on Riemannian manifolds (Tu1)

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We study the proximal point method with Bregman distances to solve the Variational Inequality Problem (VIP): given a nonempty closed convex set X in a Hadamard manifold M (complete finite dimensional simply connected Riemannian manifold with nonpositive sectional curvature), find $x^* \in X$ such that

$$\langle T(x^*), \exp_{x^*}^{-1} x \rangle \geq 0, \forall x \in X,$$

where $T : M \to TM$ is a monotone vector field on the manifold. Introducing some natural assumptions on the problem we prove the convergence of the sequence generates by the proximal algrithm to a solution of the (VIP).

Proximal methods for the resolution of variational problems (Mo1)

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We consider a variational problem which consists in the minimization of the functional $J(u) = \operatorname{ess\,sup}_{x\in\Omega} f(x, u, Du)$ where $u \in W^{1,\infty}(\Omega)$ with $v = g$ on $\partial\Omega$ and $\Omega \subset \mathbb{R}^n$ is bounded. This work deals with the numerical approximation of the solution. Given the convexity of the data functions, through a discretization procedure, we obtain a finite dimensional nonsmooth optimization problem, and we find a minimum by applying a proximal-like method. In general, solutions are not unique, but a canonical extension is given by an *absolute minimizer*, which is defined as the function that solves the problem on every subdomain of Ω. It is known that an absolute minimizer u is a viscosity solution of an Euler-Lagrange equation, the so-called Aronsson equation $A_f[u] = 0$, which in the case of optimal Lipschitz extensions, i.e. with $f(x, u, Du) = \|D(u)\|^2$, becomes the ∞-Laplace equation. We show numerical examples and we discuss some ideas for obtaining optimality conditions for absolute minimizers.

A quasi-Newton algorithm for multiobjective optimization (Th1)

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In this work we focus on a descent method for solving smooth unconstrained multicriteria optimization problems that uses the quasi-Newton approach with a globalization strategy, specifically the trust region approach. The idea of the proposed method is to mimick the good characteristics of a quasi-Newton method when applied to solve a scalar nonlinear optimizazion problem, such as the fast convergence compared to the steepest descent method and the natural global convergence as opposed to the Newton method.

An extension of Fenchel's conjugation (Mo3)

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In this work we consider an extension of Fenchel's conjugation for a particular class of proper functions. The difference between the classical Fenchel's conjugation and our extension is the bilinear part, which can be viewed as an element in a topological dual space. While Fenchel's conjugation considers only continuous linear functionals, our conjugation works with linear and nonlinear functionals. The classical Fenchel's conjugation is the special case of the extension given here. Some special examples illustrate the difference between Fenchel's conjugation and the new extension of the classical conjugation.

Approaches to risk in optimization under uncertainty (TuP4)

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Decisions often need to be made before all the facts are in. In optimization under uncertainty, this greatly influences how a problem should or should not be formulated. Many approaches with long traditions behind them can be criticized from one angle or another, and the need for better alternatives has come to be widely appreciated.

This talk will provide an overview of the issues and challenges and present a unified framework in which different approaches can be compared. Recently developed techniques involving "coherent measures of risk" will be emphasised.

A hybrid predictor-corrector-proximal-point method
for the linear monotone complementarity problem (Tu4)

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In this work we consider the linear monotone complementarity problem LMCP(F), without making any assumption on the existence of the classic central path (e.g. boundedness of the solution set). To solve this problem we propose an hybrid algorithm, which combines an interior-point algorithm (a predictor-corrector scheme for solving the LMCP(F)) with an extragradient-proximal-point scheme (Solodov-Svaiter inexact proximal point method for finding a zero of a maximal monotone operator). The algorithm is related to log-quadratic central trajectories for the

LMCP(F), which appear from considering log-quadratic barrier functions, proposed by Auslender, Teboulle and Ben-Tiba. We prove global convergence of the algorithm, present a complexity analysis and some numerical results. We also consider the extension of these ideas to other cases of interest.

Hedging under proportional transaction costs (Tu2)

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We consider a market with proportional transaction costs (e.g. between d currencies) and want to hedge a claim by trading in the underlying assets. The superhedging problem is to find the set of d-dimendional vectors of initial capital that allow to superhedge the claim. It can be shown that the functional that assigns the set of superhedging prices to a claim is nothing else than a set-valued coherent risk measure.

When starting with a vector of initial capital that does not allow to superhedge, a shortfall at maturity is possible. We will formulate and discuss the problem of finding an optimal strategy that minimizes the risk of falling short when the risk is evaluated by a set-valued risk measure.

The resulting convex optimization problem has a set-valued objective; new solution concepts will be introduced and duality for such problems will be discussed.

Bundle methods for nonconvex optimization (Th4)

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Proximal bundle methods have been shown to be highly successful optimization methods for nonsmooth convex optimization. We address the question of whether bundle methods can be extended to work for nonconvex problems. We review some past results for proximal bundle methods and demonstrate a method for extending bundle methods to a nonconvex setting. The method is based on generating cutting planes models, not of the objective function but of a local convexification of the objective function. The convexification parameter is calculated ”on the fly,” which allows for both strong convergence results and the ability to inform the user on when proximal parameters are too small to ensure a unique proximal point of the objective function. The methos is assessed by encouranging preliminary numerical results on polynomials.

On the method of Moving Asymptotes for structural optimization: a relaxed and globally convergent version (Th4)

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The Method of Moving Asymptotes (MMA) is seminal among the structural optimization community. In this work, modified versions of the MMA are proposed, based on the spectral parameter (Birgin and Martnez, 2001 and Raydan, 1997) for updating a key parameter of the model and on relaxing the conservative condition used as the acceptance criterion by Svanberg (2002).

The second-order information present in the spectral parameter is included in the model functions that define the rational approximations of the objective function and the nonlinear constraints in the beginning of each iteration, somehow improving the quality of the approximations.

The conservative condition is relaxed by means of a forcing sequence (Li and Fukushima, 2000) that goes to zero as the external iterations progress. Therefore, the conservative condition is relaxed in the beginning, and ultimately achieved in the end. The forcing sequence is controlled so that global convergence is proved to be maintained.

The computational tests were performed using a primal-dual method to solve the subproblems, as well as a trust-region algorithm applied to the dual problem.

Numerical experiments indicate that the ideas are promising. First, the computation of the spectral parameter is worth for reducing the total effort of the algorithm as compared with the original version. Moreover, driving to zero a possible violation of the fesibility indeed speeds the algorithm. As a result, the adopted relaxed conservative condition usually reduces the total number of solved subproblems, without damaging the global convergence property.

An inexact projection method for an equilibrium problem in a Hilbert space (Tu1)

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In this work we analyse an iterative method for an equilibrium problem in a Hilbert space H which involves a closed convex set C an a bifuction $f : H \times H \to (-\infty, \infty]$ with $C \subseteq dom(f)$ such that $f(x, \cdot)$ is convex and Gateux-differentiable at x, for all $x \in C$. We consider a robust iterative method in the sense that appropriately small perturbations do not alter the convergence of the algorithm. It is an implementable algorithm since at each iteration, only one projection onto a simple approximation set of C (like polyhedron) is calculated. We obtain that the sequence generated by de method has feasible weak accumulation points under the assumptions that the equilibrium problem is solvable and the function g defined as the G-differential of $f(x, \cdot)$ at x for all $x \in E \supseteq C$ is cocoercive with respect to one solution. When some continuity condition on the bifunction or on the G-differential is required, we obtain that the weak accumulation points are solutions of the equilibrium problem. Moreover, we establish that the whole sequence converge to a solution of the equilibrium problem if g is cocoercive on E respect the solution set.

An incomplete oblique projections method for solving large-scale box constrained least squares problems (Fr3)

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Large and sparse systems of linear equations or inequalities arise in many important applications, as image reconstruction from projections, radiation therapy treatments planning, computational mechanics and optimization problems. In practice, those systems are often inconsistent, and one usually seeks a point $x^* \in \mathbb{R}^n$, that must satisfy bound constraints. The aim of this paper is to extend the applicability of the IOP algorithm for solving inconsistent linear systems to the box constrained case. The key idea is to employ incomplete projections onto the set of solutions of the augmented system $Ax - r = b$, together with the box constraints. In order to compute the incomplete projections we apply our DACCIM algorithm. We will present the extended BIOP algorithm based on a scheme similar to the one of IOP, adding the conditions for accepting an approximate solution in the box. The extended algorithm converges to the minimal norm solution of the particular constrained least squares solutions. The theoretical properties of the new algorithm will be analyzed, and numerical experiences will be presented comparing its performance with some well-known methods.

Eigenvalue analysis of linear complementarity systems (ThP2)

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Equilibria in mechanics or in transportation models are not always expressed through a system of equations, but sometimes they are characterized by means of complementarity conditions involving a convex cone. This talk deals with the analysis of cone-constrained eigenvalue problems and some of their numerous applications. Among other issues, we estimate the maximal number of eigenvalues in a cone-constrained problem. Special attention is paid to the case in which an eigenvector is required to have only nonnegative entries. We also introduce and study two algorithms for solving numerically such type of eigenvalue problems.

A predictor-corrector method for the Generalized Nonlinear Complementarity Problem in polyhedral cones (Tu4)

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In this work a predictor-corrector method for solving the Generalized Nonlinear Complementarity Problem (GNCP) in polyhedral cones is proposed. Such method may be considered a member of the so-called Chebyshev-Halley family of methods for nonlinear systems, adapted to conform with the interior-point approach. Applied to a linear complementarity problem, the proposed method turns into the well-known Mehrotra's predictor-corrector method. A quadratic local convergence result is obtained with the assumptions usually made for the class of problems under consideration. Numerical experiments indicate that the ideas are promising, in the sense that for some problems the cost-benefit of computing the predictor-corrector steps is worth it for reducing the total effort of the algorithm as compared with Newton's method.

A semismooth Newton method for the continuous quadratic knapsack problem (Mo4)

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We present a semismooth Newton method for the continuous quadratic knapsack problem, that is, the projection onto the intersection of a box and a plane in $\mathbb{R}^n$. Our algorithm is derived from the minimum reformulation of the linear complementarity system associated to the KKT conditions of the original problem. We show that the Newton method does not need a globalization strategy to converge and present some numerical results comparing its performance with other algorithms recently proposed in the literature.

Identifying structure of nonsmooth convex functions by the bundle technique (ThP3)

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We consider the problem of minimizing nonsmooth convex functions, defined piecewise by a finite number of functions each of which is either convex quadratic or twice continuously differentiable with positive definite Hessian on the set of interest. This is a particular case of functions with primal-dual gradient structure, a notion closely related to the so-called VU-space decomposition: at a given point, nonsmoothness is locally restricted to the directions of the subspace V, while along the subspace U the behaviour of the function is twice differentiable. Constructive identification of the two subspaces is important, because it opens the way to devising fast algorithms for nonsmooth optimization (by following iteratively the manifold of smoothness, on which superlinear U-Newton steps can be computed). In this work we show that for the class of functions in consideration, the information needed for this identification can be obtained from the output of a standard bundle method for computing proximal points, provided a minimizer satisfies the nondegeneracy and strong transversality conditions.

An existence result for equilibrium problems with some surjectivity consequences (Tu1)

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We present conditions for existence of solutions of equilibrium problems, which are sufficient in finite dimensional spaces, without making any monotonicity assumption on the bifunction which defines the problem. As a consequence we establish surjectivity of set-valued operators of the form $T + \lambda I$, with $\lambda > 0$, where T satisfies a property weaker than monotonicity, which we call pre-monotonicity. We study next the notion of maximal pre-monotonicity. Finally we adapt our condition for non-convex optimization problems, obtaining as a by-product an alternative proof of Frank-Wolfe's Theorem.

A proximal method with separable Bregman distances for quasiconvex minimization over the nonnegative orthant (Mo1)

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We present an interior proximal method with Bregman distance, for solving the minimization problem with quasiconvex objective function under nonnegative constraints. The Bregman function is considered separable and zone coercive, and the zone is the interior of the positive orthant. Under the assumption that the solution set is nonempty and the objective function is continuously differentiable, we establish the well definedness of the sequence generated by our algorithm and obtain two important convergence results, and show in the main one that the sequence converges to a solution point of the problem when the regularization parameters go to zero.

One of aspects that motivates this work is a decision model that uses proximal methods with Bregman distances and quasiconvex objective functions. A striking behavioral model to motivate this paper is to consider agents which not only try to improve or optimize by choosing better actions, as the usual case is, but also try to learn how to be able to carry out new actions, which is costly and has huge implications on the optimizing process itself. In this application we show how a Bregman function is one of the good ways for modelling the learning cost and, more generally, a capability cost.

To describe human behaviors we need a model which includes not only "costs to carry out an action" and the "benefits of this action" (a behavioural "cost-benefit" analysis) but also need to model learning and knowledge costs i.e. "costs to learn how carry out an action", and, in a more general manner, capability costs, i.e., "costs that allow the execution of an action". Then, a traditional model of optimization such as finding the maximum $x^* \in X$ of a payoff function $g(\cdot) : x \in X \longmapsto g(x) \in R$ (utility, profit, ...) so that $\overline{g} = \sup\{g(y), y \in X\} = g(x^*)$ or the minimum of a cost function $f(\cdot) : x \in X \longmapsto f(x) \in R$ cannot adequately describe the behaviour of most agents, such as a consumer, a producer, a worker, ... unless we renounce to modelling the learning part of their activity (how they gradually build skills, competencies,...). As an agent does not succeed to do the best when one starts doing something for the first time, we need to model how one learns to carry out increasingly better actions, and more generally how one improves one's capabilities.

Projective splitting methods for the sum of monotone operators and the hybrid proximal-projection method (MoP1)

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Projective splitting methods for the sum of monotone operators and the hybrid proximal-projection method

Many relevant problems in mathematical programming can be formalized a the problem of finding a zero of a maximal monotone operator. For a generic maximal monotone operator, the proximal point method is an theoreticaly effective. It does not prescribe how each proximal subproblem shall be (approximately) solved, but it requires summable errors along the sequence of iterations. Nevertheless, the proximal point method is a general framework for the design and analysis of computational methods for solving particular instances of the inclusion problem.

By interpreting the exact proximal point method as a projection method, an inexact proximal point method with a relative error tolerance can be easily obtained. Moreover, this approach unifies Korpolevich's method and the proximal point method.

Very often, a mathematical programming problem can be formulated as the problem of finding a zero of a sum of maximal monotone operators, where each of the operators has a easily computable resolvent. For this problem, decomposition methods provide algorithms for solving the inclusion problem which deals with each operator separately. Surprisingly, here also projection methods can be used, provided one accept to work in a enlarged space.

Application of parametric optimization techniques for optimal control (Tu3)

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Parametric optimization offers very useful techniques for solving optimization problems in finite-dimensional spaces whenever the objective function (or other entries) continuously depends on some unknown parameter. These techniques yield as a result a minimizing (or maximizing) curve that depends continuously on the original parameter.

On the other hand, traditional techniques for solving optimal control problems rely on finding the *nominal control trajectory* that minimizes the Hamiltonian at each instant along the time segment. Such trajectory should be continuously time-dependent as well. Basing on this argument, we can establish a bond between these two types of optimization techniques and will finally show how to apply the parametric optimization techniques for solving optimal control problems.

Effectively, the nominal optimal control trajectory can be obtained as a solution curve of the one-parametric nonlinear optimization problem by means of so-called *pathfollowing methods* (also known as continuation or homotopy methods).

The proposed technique will be also suitable for an amplified class of admissible control sets, since the existent methods for finding the nominal optimal control trajectory tend to be individual for some specific types of control constraints (such as polyhedron constraints or a set of isolated points).

For numerical implementation of the proposed technique, as well as for simulations, the authors make use of the software package *PAFO* (*PA*th *FO*llowing).

Preconditioned Iterative Methods for solving indefinite problems (FrP3)

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The indefinite problem appears in constrained optimization, numerical PDE and other applied areas. Preconditioned iterative methods for solving the problem are an active research issue in both the optimization and the numerical linear algebra community. Here some properties of the indefinite systems are reviewed. Then, basic iterative methods and some preconditioners are presented.