



# Workshop on Conservative Dynamics and Symplectic Geometry

August 03<sup>rd</sup> to 07<sup>th</sup>, 2009  
IMPA, Rio de Janeiro, Brazil

## General Information

INSTITUTO NACIONAL DE MATEMÁTICA PURA E APLICADA

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## **Welcome to Workshop on Conservative Dynamics and Symplectic Geometry**

The Workshop aims to gather experts on Conservative Dynamics and Symplectic Geometry, from Brazil and abroad, in a leisurely ambiance.

Just as in the first two editions of the meeting, held at IMPA in 2005 and 2007, the program includes a small number of lectures leaving plenty of time for informal discussions.

The main topics are Symplectic and Contact Geometry, Floer and Contact Homology, Dynamics of Lagrangian Systems and Twist Maps, Hamiltonian Dynamics and Integrable Systems, Aubry-Mather theory, among others.

### **Organizing Committee**

Henrique Bursztyn (IMPA)

Leonardo Macarini (UFRJ)

Marcelo Viana (IMPA)

### **Participants**

Alberto Abbondandolo (Pisa)

Miguel Abreu (Lisbon)

Silvia Anjos (Lisbon)

Leo Butler (Edinburgh)

Gonzalo Contreras (CIMAT)

Mario Jorge Dias Carneiro (UFMG)

Albert Fathi (ENS-Lyon)

Urs Frauenfelder (Munich)

Viktor Ginzburg (Santa Cruz)

Umberto Hryniewicz (UFRJ)

Renato Iturriaga (CIMAT)

Dan Jane (IMPA)

Ely Kerman (Illinois)

Boris Khesin (Toronto)

Otto van Koert (Hokkaido)

Artur Lopes (UFRGS)

David Martinez (Lisbon)

Eva Miranda (Barcelona)

Al Momin (Leipzig)

Alexandru Oancea (Strasbourg)

Gabriel Paternain (Cambridge)

Diogo Pinheiro (University of Lisbon)

Clodoaldo Ragazzo (USP)

Tudor Ratiu (Lausanne)

Rafael Ruggiero (PUC-RJ)

Pedro Salomao (USP)

Carlos Tomei (PUC-RJ)

Salvador Zanata (USP)

N-T Zung (Toulouse)

# Program

## Auditorio Ricardo Mañe

Hour	Monday 03	Tuesday 04	Wednesday 05	Thursday 06	Friday 07
09:30	Registration				
10:00					
10:00	Ratiu	Paternain	Ginzburg	Miranda	Lopes
11:00					
11:00			Coffee		
11:30	Abbondandolo	Abreu	Frauenfelder	Kerman	Pinheiro
12:30					
12:30			Lunch		
14:00	Butler	Oancea		Hryniewicz	Zung
15:00					
15:00		Coffee		Coffee	
15:30	Contreras	Van Koert	FREE	Salomão	Fathi
16:30					
17:00	Khesin	Saldanha		Momin	
18:00					
18:00		Cocktail			
20:00					

## Available Abstracts

### Alberto Abbondandolo (Pisa)

**Título:** Chain level computations in Rabinowitz-Floer homology

**Resumo/Abstract:**

Rabinowitz-Floer homology is the homology of a chain complex associated to the free period Hamiltonian action functional. The Rabinowitz-Floer homology of the unit cotangent disk bundle can be determined from a long exact sequence discovered by K. Cieliebak, U. Frauenfelder, and A. Oancea. In this talk I will explain how this long exact sequence can be understood from a chain level point of view, starting from the cellular filtration of the free loop space induced by the geodesic energy functional. This construction generalizes to fiberwise convex domains in a cotangent bundle which contain a Lagrangian graph.

### Miguel Abreu (Lisbon)

**Título:** Contact Homology of Toric Contact Manifolds

**Resumo/Abstract:**

In this talk I will introduce toric contact manifolds and present their classification due to Banyaga-Molino, Boyer-Galicki and Lerman (2003). I will then describe why and how their contact homology, a powerful invariant of contact manifolds, can be explicitly computed. As a particular application I will show that a family of contact structures arising in the work of Gauntlett-Martelli-Sparks-Waldram (2004), a group of mathematical-physicists, contains infinitely many new inequivalent contact structures on  $S^2 \times S^3$ .

This is joint work with Leonardo Macarini.

### Leo Butler (Edinburgh)

**Título:** Exotic tori and complete integrability

**Resumo/Abstract:**

In hamiltonian mechanics, a key role is played by completely integrable systems. A priori, the existence of such a system is a smooth invariant of the configuration space.

This talk will demonstrate that this invariant is non-trivial. We will show that, if a smooth manifold homeomorphic to the  $n$ -torus admits a completely integrable convex hamiltonian, then it is actually diffeomorphic to the standard  $n$ -torus. On the other hand, we will exhibit certain families of topological 7-manifolds whose 28 smooth structures each admit a completely integrable convex hamiltonian.

### Gonzalo Contreras (CIMAT)

**Título:** Existence of periodic orbits for convex Lagrangians

**Resumo/Abstract:**

We prove that an autonomous convex superlinear lagrangian system has a periodic orbit in almost every energy level.

**Albert Fathi (ENS-Lyon)**

**Título:** Smoother critical subsolutions to Hamilton-Jacobi Equation

**Resumo/Abstract:**

We will show that a critical subsolution of the Hamilton-Jacobi Equation which is  $C^k$  in a neighborhood of the Aubry set can be replaced by a critical subsolution which is  $C^k$  everywhere. This generalizes a result of Patrick Bernard.

We will give an introduction to the problem, and after that explain the rather short proof.

**Urs Frauenfelder (Munich)**

**Título:** On Rabinowitz Floer homology

**Resumo/Abstract:**

Rabinowitz Floer homology is the semiinfinite dimensional Morse homology associated to a Lagrange multiplier functional which played already a major role in the work of Rabinowitz. Applications of this new homology include existence and multiplicity results for leafwise intersection points, obstructions to exact contact embeddings, and the characterisation of Mane's critical value. In this talk I explain the basics of Rabinowitz Floer homology.

**Viktor Ginzburg (Santa Cruz)**

**Título:** Hamiltonian Systems With Finitely Many Periodic Orbits

**Resumo/Abstract:**

In this talk we discuss the narrow but interesting class of Hamiltonian systems with finitely many periodic orbits. We show that, under suitable additional hypotheses on the ambient manifold, the action and mean indexes of periodic orbits must satisfy certain relations for such a system. Then, considering now general Hamiltonian systems, we use this fact to prove the generic existence of infinitely many periodic orbits. The talk is based on joint works with Basak Gurel and Ely Kerman.

**Umberto Hryniewicz (UFRJ)**

**Título:** Disk-Like Global Surfaces of Section on Convex 3-Dimensional Energy Levels

**Resumo/Abstract:**

Let  $M$  be the boundary of a smooth, bounded, generic, strictly convex domain in 4-dimensional euclidean space. We present necessary and sufficient conditions for a closed Hamiltonian orbit on  $M$  to bound a disk-like global surface of section.

**Ely Kerman (Illinois)**

**Título:** Hofer's geometry, geodesic flows and Maslov class rigidity

**Resumo/Abstract:**

A path of Hamiltonian diffeomorphisms which does not minimize the Hofer length functional must have periodic orbits with special properties. These orbits play an important role in several symplectic rigidity phenomena. In this talk I will describe the how they can be used to determine new restrictions on the Maslov class of displaceable Lagrangian submanifolds. In particular, these restrictions are obtained by considering the length minimizing properties of Hamiltonian flows which are supported near the Lagrangian submanifold and are reparameterizations of geodesic flows on it.

**Boris Khesin (Toronto)****Título:** A nonholonomic Moser theorem and diffeomorphism groups**Resumo/Abstract:**

We discuss the following nonholonomic version of the classical Moser theorem: given a bracket-generating distribution on a connected compact manifold (possibly with boundary), two volume forms of equal total volume can be isotoped by the flow of a vector field tangent to this distribution.

We also present the Hamiltonian framework for the corresponding mass transport problem as an infinite-dimensional Hamiltonian reduction on diffeomorphism groups. The subriemannian heat equation turns out to be a gradient flow on the "nonholonomic" Wasserstein space with the potential given by the Boltzmann relative entropy functional. (This is a joint work with Paul Lee.)

**Otto Van Koert (Hokkaido)****Título:** Linearized Contact Homology of Connected Sums**Resumo/Abstract:**

Contact homology is an invariant of contact manifolds defined by counting holomorphic curves in a symplectization. I will give an overview of several versions of contact homology and discuss joint work with Frederic Bourgeois on connected sums.

For connected sums of contact manifolds one can show that many holomorphic curves cannot exist. This can then be used to show that there is a long exact sequence for the linearized contact homology of connected sums of contact manifolds.

**Artur Lopes (UFRGS)****Título:** Piecewise Analytic Subactions for Analytic Dynamics**Resumo/Abstract**

We consider a piecewise analytic expanding map  $f : [0, 1] \rightarrow [0, 1]$  of degree  $d$  which preserves orientation, and an analytic positive potential  $g : [0, 1] \rightarrow \mathbb{R}$ .

We address the analysis of the following problem: for a given analytic potential  $\beta \log g$ , where  $\beta$  is a real constant, it is well known that there exists a real analytic (with a complex analytic extension to a small complex neighborhood of  $[0, 1]$ ) eigenfunction  $\phi_\beta$  for the Ruelle operator. One can ask: what happens with the function  $\phi_\beta$ , when  $\beta$  goes to infinity? The domain of analyticity can change with  $\beta$ . The correct question should be: is  $\frac{1}{\beta} \log \phi_\beta$  analytic in the limit, when  $\beta \rightarrow \infty$ ? Under a uniqueness assumption, this limit, when  $\beta \rightarrow \infty$ , is in fact a calibrated subaction  $V$  (see below definition). We show here that under certain conditions and for a certain class of generic potentials this continuous function is piecewise analytic (but not analytic). In a few examples one can get that the subaction is analytic (we need at least to assume that the maximizing probability has support in a unique fixed point).

The following question is related to the above problem. Denote

$$m(\log g) = \max_{\nu \text{ an invariant probability for } f} \int \log g(x) d\nu(x),$$

and  $\mu_\infty$  any probability which attains the maximum value. Any one of these probabilities  $\mu_\infty$  is called a maximizing probability for  $\log g$ . We assume here that the maximizing probability is unique.

The probability  $\mu_\infty$  is the limit of the Gibbs states  $\mu_\beta$ , for the potentials  $\beta \log g$ . In this sense one can say that  $\mu_\infty$  corresponds to the Statistical Mechanics at temperature zero.

In order to analyze ergodic properties of such probability  $\mu_\infty$ , it is natural to associate to such  $f$  a bijective transformation  $\hat{\sigma}$ , which acts on  $\hat{\Sigma} = \Sigma \times [0, 1]$ , where  $\Sigma = \{1, 2, \dots, d\}^{\mathbb{N}}$ .

One can consider  $W$  the involution kernel associated to  $\log g$ , where  $W : \hat{\Sigma} \rightarrow \mathbb{R}$ , and  $W(w, x)$  is defined for all  $w \in \Sigma$  and  $x \in [0, 1]$ .

We show the existence of an analytic involution kernel for  $\log g$  (in the sense that it is analytic in the second variable, for  $w$  fixed) and a interesting relation with the dual potential  $(\log g)^*$  defined in the Bernoulli space  $\Sigma$ .

Using the above results we show that when  $\mu_\infty$  is unique, has support in a periodic orbit, the analytic function  $g$  is **generic** and satisfies the **twist condition**, then the calibrated sub-action  $V : [0, 1] \rightarrow \mathbb{R}$  for the potential  $\log g$  is piecewise analytic. By definition, the calibrated subaction is the function  $V$  such that

$$\sup_{y \text{ such that } f(y) = x} \{ V(y) + \log g(y) - m(\log g) \} = V(x).$$

We assume the twist condition only in some of the proofs.

An interesting case where the theory can be applied is when  $\log g(x) = -\log f'(x)$ . In this case we relate the involution kernel to the so called scaling function.

### **Eva Miranda (Barcelona)**

**Título:** Rigidity for Hamiltonian actions in Symplectic and Poisson

**Resumo/Abstract:**

The theorem of Liouville-Mineur-Arnold for integrable systems on symplectic manifolds entails rigidity in the integrable category for those systems. In fact, action-angle coordinates give a natural action of a torus by translations on the symplectic manifold and integrable systems with regular compact leaves constitute a basic example of semilocal Hamiltonian actions of tori. This rigidity result connects to classical rigidity results for actions of compact Lie groups on compact smooth manifolds (Palais).

In this talk the main question under consideration is whether we can export this theorem of Palais for general actions of compact Lie groups/algebras to the Symplectic and Poisson setting. In the Symplectic case this can be done via the path method. In the Poisson setting, we can still get a rigidity result for semisimple Hamiltonian actions of compact type at the local (fixed point), semilocal (compact invariant submanifold) and global (compact) levels. The price to pay to go from the Symplectic to the Poisson world is the use of more sophisticated techniques like Nash-Moser. This result can be seen as an "infinitesimal rigidity implies rigidity" theorem in the spirit of Mather.

Most of the results presented in this talk are joint work with Philippe Monnier and Nguyen Tien Zung.

### **Alexandru Oancea (Strasbourg)**

**Título:** Reeb chords for fillable Legendrians

**Resumo/Abstract:**

I will give quantitative results on the number of Reeb chords with endpoints on a Legendrian submanifold  $L$  of a contact manifold  $M$ , in the situation where  $M$  admits a symplectic filling, and  $L$  admits a Lagrangian filling. The main tool that I will use is wrapped Floer homology.

**Gabriel Paternain (Cambridge)****Título:** Symplectic topology of Mane's critical values**Resumo/Abstract:**

Abstract: Consider a closed Riemannian manifold  $M$  and let  $\sigma$  be a closed 2-form whose pull-back to the universal covering of  $M$  is exact. I will discuss the changes in the symplectic topology of a hypersurface  $|p|^2=2k$  in the twisted cotangent bundle determined by  $\sigma$  as  $k$  makes its transition from high energies to low energies. It has been known for some time (Aubry-Mather theory) that drastic changes in the dynamical properties of the hypersurface take place at the Mane's critical values. I will try to relate these phase transitions to symplectic properties like displacement, stability and vanishing of the Rabinowitz Floer homology. This is joint work with Kai Cieliebak and Urs Frauenfelder.

**Diogo Pinheiro (University of Lisbon)****Título:** An asymptotic universal focal decomposition for Non-isochronous potentials and some consequences**Resumo/Abstract:**

ABSTRACT. Galileo, in the XVII century, observed that the small oscillations of a pendulum seem to have constant period. In fact, the Taylor expansion of the period map of the pendulum is constant up to second order in the initial angular velocity around the stable equilibrium. It is well known that, for small oscillations of the pendulum and small intervals of time, the dynamics of the pendulum can be approximated by the dynamics of the harmonic oscillator. We study the dynamics of a family of mechanical systems that includes the pendulum at small neighbourhoods of the equilibrium but after long intervals of time so that the second order term of the period map can no longer be neglected. We characterize such dynamical behaviour through a renormalization scheme acting on the dynamics of this family of mechanical systems. The main theorem states that the asymptotic limit of this renormalization scheme is universal: it is the same for all the elements in the considered class of mechanical systems. As a consequence we obtain an universal asymptotic focal decomposition for this family of mechanical systems. Furthermore, we obtain that the asymptotic trajectories have a Hamiltonian character and compute the action of each element in this family of trajectories. We conclude with a description of the utility that the asymptotic universal focal decomposition may have in the computation of propagators in semiclassical physics.

**Tudor Ratiu (Lausanne)****Título:** Integrable flows on the symplectic group and optimal control**Resumo/Abstract:**

In this talk we explore two phenomena related to the free rigid body equations on semisimple Lie algebras. The first is a new system on symmetric matrices introduced by Bloch and Iserles. Its Liouville integrability is proved and the flow is linearized in the generic case. This system is isomorphic to the free rigid body on the symplectic group. The second is a symmetric representation of the  $SO(N)$  rigid body.

The explanation of this formulation of the equations of motion resides in a new class of optimal control problems whose cost function is the energy of the system and the constraining dynamics is given by the infinitesimal generator of its symmetry algebra.

**Nicolau C. Saldanha (PUC-RJ)**

**Título:** Examples of dynamics of eigenvalue computation

**Resumo/Abstract:**

Wilkinson's iteration is one of the most popular algorithms to compute eigenvalues of a real symmetric tridiagonal matrix. This scenario leads to several different phase spaces, with corresponding somewhat incompatible structures. In this talk we give a survey of these points of view and of some results concerning the rate of convergence. Joint work with Ricardo Leite and Carlos Tomei.

**Pedro Salomão (USP)**

**Título:** On the existence of openbook decompositions for Reeb dynamics on the tight 3-sphere

**Resumo/Abstract:**

We give necessary and sufficient conditions for the existence of openbook decompositions with disk-like pages for Reeb dynamics on the tight 3-sphere, assuming all closed orbits are non-degenerate. It is a joint work with U. Hryniewicz (UFRJ).

**N-T Zung (Toulouse)**

**Título:** On the symplectic geometry of Gelfand-Cetlin system

**Resumo/Abstract:**

TBA

## **WELCOME TO IMPA!**

### **1) Computational Facilities:**

The participants can use the computers on the Hall of the 2nd floor

- The login is *impa2009*
- The password is *castorina110*

We also have wireless (WIFI) connection. The ESSID of the network is *impa-wl* and the password is *impacastorina*.

### **2) Location of IMPA:**

IMPA is located near the Botanical Garden in the city of Rio de Janeiro, Brazil.

The address is:

Estrada Dona Castorina, 110 – CEP: 22460-320 Rio de Janeiro, RJ – Brasil.

However, many cab drivers may need the following instructions which we reproduce in Portuguese:

***HORTO, POR FAVOR: Vou para o IMPA na  
ESTRADA DONA CASTORINA, 110  
NO FINAL da rua PACHECO LEÃO à DIREITA DEPOIS do PONTO  
FINAL da LINHA DE ONIBUS 409.***

### **3) Public Transportation:**

How to get to IMPA:

The easiest and most cost effective (timewise) way to get to IMPA is by cab as described above.

However, if you prefer to use public transportation see the next paragraph.

- From Copacabana:

You can use the bus line 125 (Jardim Botânico) from Avenida Princesa Isabel or Rua Barata Ribeiro and get off at the final stop. You should then walk uphill to Estrada Dona Castorina; IMPA is on the right hand side.

Since the 125 bus is somewhat infrequent, it is usually faster to follow a different route. Take the 572 or 584 bus and get off on Rua Jardim Botânico at the stop near ABBR and the “Pão de Açúcar” supermarket. Then, walk to Rua Lopes Quintas (which crosses Rua Jardim Botânico), go to the bus stop near the newsstand, take a 409 or 125 bus and get off at the final stop. From then on, follow the instructions at the end of the previous paragraph.

- From Ipanema and Leblon

You can use the bus line 125 (to Jardim Botânico) from Rua Prudente de Moraes (Ipanema), Avenida General San Martin (Leblon), or Avenida Bartolomeu Mitre (Leblon) and get off at the final stop. You should then walk uphill to Estrada Dona Castorina; IMPA is on the right hand side.

Since the 125 bus is somewhat infrequent, it is usually faster to follow a different route. Take the 572, 512 or 584 bus and get off on Rua Jardim Botânico at the stop near ABBR and the “Pão de Açúcar” supermarket. Then, walk to Rua Lopes Quintas (which crosses Rua Jardim Botânico), go to the bus stop near the newsstand, take a 409 or 125 bus and get off at the final stop. From then on, follow the instructions at the end of the previous paragraph.

- From Flamengo, Botafogo and Humaitá

You can use the bus line 409 (Sans Pena – Horto) from Praia do Flamengo (Flamengo Beach), Praia de Botafogo (Botafogo Beach) or Rua Humaitá, and get off at the final stop. Then walk uphill to Estrada Dona Castorina; IMPA is on the right hand side.