Generalized dimensions and complexity functions of infinite sequences

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Abstract

Given an infinite sequence over a finite alphabet with b letters (which can be identified with a real number written in basis b), its complexity function is a function p defined on the set N of the positive integers such that p(n) is the number of subwords of lenght n of our infinite sequence. It is a measure of the complexity of the subshift on the closure of the orbit of the sequence by the usual unilateral shift. Given a function f(n), we consider the set of real numbers with complexity bounded by f. It has positive Hausdorff dimension if and only if f grows exponentially fast. We study this sets in general (which, in the more interesting cases, have zero Hausdorff dimension), and estimate generalized fractal dimensions of them, associated to more general gauge functions than the functions x^s, $s>0$ (which are used to define the Hausdorff dimension).