

Maximizing measures, subactions, large deviations

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Consider a α -Hölder function $A : \Sigma \rightarrow \mathbb{R}$ and a dynamical system $F : \Sigma \rightarrow \Sigma$, where (Σ, d) is a metric space. Denote by $\mathcal{M}(F)$ the set of invariant probabilities.

A A -maximizing measure μ_A is a probability which satisfies

$$\sup\left\{\int Ad\nu \mid \nu \in \mathcal{M}(F)\right\} = m(A) = \int Ad\mu_A.$$

A subaction for A is a real continuous function $U : \Sigma \rightarrow \mathbb{R}$ such that

$$A \leq U \circ \sigma - U + m(A)$$

everywhere on Σ .

The subaction is a natural tool for analyzing such maximizing probability. The inequality is an equality on the Aubry set.

We will describe several new results, mainly for expanding systems, about problems related to the existence and other properties of A -maximizing probabilities. The results are in collaboration with several different people: Ph. Thieullen, C. Contreras, R. Ruggiero, A. Baraviera, etc...

I) Generically are unique and concentrated on a periodic orbit.

II) Generically for expanding $F : S^1 \rightarrow S^1$ of class $(C^{1+\alpha})^+$, the one maximizing $\int \log |F'(x)| d\nu(x)$ is concentrated on a periodic orbit.

III) Similar results for Anosov diffeos and Anosov Flows. Related results from other authors in the case of weakly expanding transformations, etc...

IV) For each $\beta \geq 0$, we denote μ_β , the unique equilibrium measure for pressure associated to βA . We show that μ_β satisfies a Large Deviation Principle for μ_A , that is, for any cylinder C of Σ (the Bernoulli space $\{1, 2, \dots, d\}^{\mathbb{N}}$),

$$\lim_{\beta \rightarrow +\infty} \frac{1}{\beta} \log \mu_\beta(C) = - \inf_{x \in C} I(x)$$

where I is explicitly defined.

V) Existence of a W kernel that relates past and future on $\{1, 2, \dots, d\}^{\mathbb{Z}}$, connecting eigenfunctions of the Ruelle operator for A to eigenmeasures of the dual of Ruelle operator for the dual potential A^* .

VI) Results relating Mather Theory and maximizing probabilities for the geodesic flow subject to homological constraints.

VII) Eigenvalues of Laplacian in negative compact constant curvature and eigenvalues of complex Ruelle operator acting on the boundary of Poincaré disk.