# Living in dimension 1 under the class $C^2$

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Starting from the classical Denjoy's Theorem, 1-dimensional dynamics has been developed assuming (in general) a $C^2$ regularity hypothesis. Technically, this is related to the possibility of controlling distortions by using the finitness of the total variation of derivatives. When the dinamics is supposed a priori to be "hyperbolic", some classical results hold already in class $C^{1+\alpha}$ for any $\alpha > 0$.

In this talk I will explain some rigidity phenomena for the dynamics which appear in class $C^{1+\alpha}$ for very precise values of $\alpha$. For instance, it can be proved (resp. it is conjectured) that for $\alpha > 1/2$ (resp. for $\alpha \geq 1/2$) the centralizer in $Diff_+^{1+\alpha}(S^1)$ of a $C^{1+\alpha}$ Denjoy conter-example cannot contain an element having irrational rotation number which is independent (over the rationals) to the rotation number of the original diffeomorphism. Moreover, this result is sharp, in the sense that for every $\alpha < 1/2$ one can construct counter-examples.

I will discuss many other results involving groups acting by diffeomorphisms on the interval or the circle showing a relationship between the algebraic structure of the group (or the combinatorial structure of the dynamics) with a precise intermediate regularity class. Quite surprisingly, the methods involved in the proofs are of probabilistic nature: it is possible to control distortions for a random product of elements in the group, and the best regularity in which this can be achieved is a kind of characteristic exponent of the random walk which is related to some algebraic feature of the group.