A New Construction Technique for Lattices from Subfields of $\mathbb{Q}(\zeta_{pq})$

Trajano Pires da Nóbrega Neto

Abstract

Trabalho conjunto com J. Carmelo Interlando, André Luiz Flores. We provide a new method to evaluate the center density of ideals in the ring of algebraic integers of subfields of $\mathbb{Q}(\zeta_{pq})$, where p and q are distinct prime numbers. This method allows us to reproduce rotated versions of known dense lattices in some dimensions. For example, we obtain lattice E_8 from several fields $\mathbb{Q}(\zeta_{pq})$. Because of their high diversity, signal constellations constructed from these dense lattices perform well on both Gaussian and Rayleigh fading channels. One application of these constellations is in mobile communications, where one single modulation/demodulation device can be used to communicate over both terrestrial and satellite links.

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The theory of algebraic lattices has shown to be extremely useful in Information Theory. Signal sets from dense lattices perform well over an additive white Gaussian channel (AWGN). In fact, Conway and Sloane [2] have shown that lattices satisfying the Minkowski bound are equivalent to codes which attain channel capacity. This establishes a link between sphere-packing and Information Theory.

In [3], Giraud and Belfiori proposed a technique for constructing signal sets suitable for the Rayleigh fading channel. The basic idea was to use lattice rotations to increase diversity, that is, the number of different values in the components of any two distinct points of the constellation. In [1], Boutros *et al.* constructed rotated versions of lattices D_4, K_{12}, and Λ_6 via ideals of $\mathbb{Q}(\zeta_n)$, for $n = 8, 21$ and 40, respectively. The principal purpose of the work was to obtain constellations having good performance in both AWGN and Rayleigh fading channels.

In this paper, starting from suitable ideals in subfields of $\mathbb{Q}(\zeta_{pq})$, we construct new rotated versions of dense lattices, for example, Λ_{24}, K_{12}, and E_8. We conjecture the existence of a lattice in dimension 28 with center density equal to 1. As in [1], the lattices presented here perform well over Gaussian and fading channels. This is particularly useful when transmitting information over terrestrial and satellite links. The same modulation/demodulation device can be used to communicate over them both.

References

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