On rings which are sums of two Pi rings: a combinatorial approach

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Abstract

We study the following open question: if a ring R is a sum of two subrings A and B both satisfying a polynomial identity, does R itself satisfy a polynomial identity? We give a positive answer to this question in case A or B is a Lie ideal or R satisfies a special "mixed" identity or $(AB)^k \subset A$ for some $A \geq 1$. Our approach is based on a comparative analysis of the sequences of codimensions of the three rings. As a reward we obtain a bound on the degree of a polynomial identity satisfied by R as a function of the degree of an identity satisfied by A and B.