On the density of algebraic foliations without algebraic invariant sets

S. C. Coutinho

This talk will be a report on joint work with J. V. Pereira.

The study of holomorphic foliations over projective varieties can be traced back to the work of G. Darboux and H. Poincaré in the 19th century. In the late 1970s, J. P. Jouanolou reworked and extended the work of Darboux in the algebraic geometric framework provided by Grothendieck. One of the key results of Jouanolou's celebrated monograph of 1979 states that a very generic holomorphic foliation of the projective plane, of degree at least 2, does not have any invariant algebraic curves. Recall that a property P holds for a *very generic point* of a variety V if the set of points on which it fails is contained in a countable union of hypersurfaces of V. Jouanolou's result has been extended in various ways by A. Lins Neto, M. Soares, V. Lunts and L. G. Mendes. In this talk I will discuss a generalization of Jouanolou's result for one dimensional foliations over any smooth projective variety.

Theorem 0.1. *Let $k \gg 0$ be an integer, and let f be a very generic global section of the twisted tangent sheaf $\Theta_X(k)$. The foliation of X determined by f has no proper invariant algebraic subvarieties of non-zero dimension.*

We have also generalized this result to fields of m-vectors, also called Pfaff equations. As an application of the theorem, we prove the following *dynamical* characterization of ampleness when X is a surface.

Theorem 0.2. *A line bundle $\mathcal{L}$ on a smooth projective surface X is ample if, and only if, $\mathcal{L}^2 > 0$ and there exists a positive integer k such that the generic section of $\Omega^1_X \otimes \mathcal{L}^{\otimes k}$ induces a foliation of X without invariant algebraic curves.*