Groups, nonassociative algebras and vertex operator algebras

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Abstract

The speaker will discuss some of his recent work on automorphism groups of vertex operator algebras (VOAs) and how nonassociative algebras come up in the degree 2 part under the first product.

The subject of VOAs is relatively new and is a complex algebraic world. For example, we have only a little progress on aspects of the theory such as understanding the simple VOAs and properties of automorphism groups. In the study of automorphisms, it is a pleasure to find links with Lie theory and with the theory of finite simple groups. The automorphism group of a finitely generated VOA is (isomorphic to) an algebraic group, which can range from a finite group to a big Lie group.

Suppose that the VOA $V = \oplus_{n \geq 0} V_n$ satisfies $dim(V_0) = 1$. Then $(V_1.0^{th})4$ is a Lie algebra. If moreover $V_1 = 0$, then $(V_2, 1^{st})$ is a commutative algebra, often not associative. (Miyamoto began calling such a commutative algebra a Griess algebra, and the name seems to have stuck. We shall discuss aspects of such finite dimensional algebras, which include some simple Jordan algebras, the 196884-dimensional algebra associated to the Monster and so on.

In the 70s, when our picture of the finite simple groups was less clear, there was a study of finite groups as automorphisms of finite dimensional commutative nonassociative algebras, as an attempt to develop a general theory of sporadic groups. There was a path to VOAs via the Monster simple group and its nonassociative algebra of dimension 196884. Recently, some ideas about groups and finite dimensional nonassociative algebras reactivated in studies of automorphism groups of VOAs. There are also connections with the work of the speaker and others on the classification of finite subgroups of exceptional Lie groups, G2, F4, E6, E7 and E8.