

Two-sided gluings of tilted algebras

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Abstract

Let A be an artin algebra. We are interested in studying the representation theory of A , thus in characterizing A by properties of the category $\text{mod}A$ of finitely generated right A -modules. One method to achieve this goal is to start from a class of algebras whose representation theory is considered to be sufficiently well-understood, and then to generalize this class to another whose representation theory is close enough to that of the preceding class. Thus, tilted algebras were introduced in [8] as a generalization of hereditary algebras. The class of tilted algebras is now considered to be one of the most useful for the general theory. For instance, it is known that an indecomposable module over an arbitrary algebra which does not lie in an oriented cycle of non-zero non-isomorphisms, is a module over a tilted algebra [9]. It was therefore natural to consider various generalizations of this notion. Thus, over the years, the following classes of algebras were defined and investigated: the quasi-tilted (which generalize the tilted and the canonical algebras of [9]) [7], the shod algebras (which generalize the quasi-tilted) [4], the weakly shod algebras (which generalize the shod and the representation-directed algebras) [5, 6] and the left and the right glued algebras (which generalize the tilted and the representation-finite algebras) [1]. The purpose of the present paper is to introduce a new class of algebras which generalizes all the previous classes.

We define an artin algebra A to be a *laura algebra* if all but at most finitely many non-isomorphic indecomposable A -modules are such that all their predecessors have projective dimension at most one, or all their successors have injective dimension at most one. We start by giving various examples and characterizations of laura algebras. We then study the representation theory of laura algebras, and our main theorem gives a full description of the Auslander-Reiten quiver of a laura algebra. The class of laura algebras is then characterized in the spirit of [1] as a double gluing of tilted algebras. Since, in general, laura algebras are representation-infinite, a measure of the complexity of the module category is given by the nilpotency of the infinite radical. We show that, if A is a representation-infinite laura algebra with nilpotent infinite radical, then its nilpotency index lies between 3 and 5, inclusively.

For further results on laura algebras, we refer the reader to [2, 3].

References

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