COLLINEATION GROUPS OF TRANSLATION PLANES AND LINEAR GROUPS

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Main Problem. Which non abelian finite simple groups can be collineation groups for a finite translation plane. (See, for example, [5].)

we can apply representation theory to the action of the group on the affine points, and permutation group theory to the action of the group on the points on the line of infinity. The collineation group of a translation plane is a semi-direct product of the translation group and the translation complement, which is a semi-linear transformation group. The subgroup of all linear transformations in the translation complement is the linear complement. Perfect subgroups of the translation complement are in the linear complement.

Two types of collineations: affine perspectivities and Baer elements attract most attention. These collineations occur in a simple collineation group in the following way. Being simple, the group is in the linear complement and it does not contain any central homology. Thus perspectivities are affine perspectivities. If the characteristic is odd, then involutions are Baer. (See, for example, [2].) If characteristic is even, then an involution is either a Baer involution or an elation with its axis a fiber. In both cases, the dimension of the set of fixed points is half the dimension of the underlying vector space. This inspires Theorem A on linear groups, and Theorems B and C on collineation groups in [3]. Besides these we will present new results concerning elements of order 4 in [4].

References

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