Abstracts dos Palestrantes:

Anton A. Duchkov
Purdue University

Continuation In Seismic Migration And Migration Velocity Analysis

Abstract. Seismic reflection data, in the single scattering or Born approximation, are commonly modelled by an integral operator mapping a medium contrast (containing reflectors), given a background medium (velocity model), to a wavefield (containing reflections). Imaging of seismic reflection data is then described by the adjoint of this integral operator (both are FIOs). We work with the class of FIOs the canonical relations of which are graphs, and which are invertible. This class has a the principal fiber bundle (and Lie group) structure. The base space is formed by the canonical transformations. Continuation can then be formalized as a section of this principal fiber bundle. Canonical transformations may be identified with contact transformations. The notion of continuation can be introduced: The continuation of an image following a path of background media without remigrating the data, or the continuation of data following a path of acquisition
geometries without demigrating an image. Continuation of image in background velocity can be exploited in developing a method for determining it (migration velocity analysis).
After continuation operators are defined satisfying a minimal set of requirements, these operators can be viewed as solution operators to pseudodifferential evolution equations (locally). Thus, continuation operators attain the form of propagators. The evolution equation leads to the introduction of continuation bicharacteristics which describe the propagation of singularities by continuation. Globally continuation can be described by finitely many evolution operators. We represent continuation as the composition of modelling and imaging FIOs selected from one-parameter family of FIOs in the above mentioned class (the parameter becomes an evolution parameter). Then we establish the explicit relation between the phase function in the kernel representation of migration FIO and the pseudodifferential operator symbol appearing in the continuation evolution equation. The local Hamiltonian that generates the continuation bicharacteristics is expressed in terms of this generating function. Center for Computational and Applied Mathematics, Purdue University, 150 N. University Street, West Lafayette, IN 47907-2067
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Antonio Leitão
UFSC

Kaczmarz methods for regularizing nonlinear ill-posed equations

Abstract:
In this talk we develop and analyze novel iterative regularization techniques for the solution of systems of nonlinear ill--posed operator equations. The basic idea consists in considering separately each equation of this system and incorporating a loping strategy.

The first technique is a Kaczmarz--type method, equipped with a novel stopping criteria. The second method is obtained using an embedding strategy, and again a Kaczmarz--type approach. We prove well-posedness, stability and convergence of both methods.

Alberto Mercado-Saucedo (Universidad de Chile)
joint work with Lucie Baudouin† and Axel Osses‡.

An inverse problem for the transmission wave equation.

We consider a transmission wave equation in two embedded domains in R2, where the speed is a1 > 0 in the inner domain and a2 > 0 in the outer domain. We prove a global Carleman inequality for this problem under the hypothesis that the inner domain is strictly convex and a1 > a2. As a consequence of this inequality, the Lipschitz stability are obtained for the inverse problem of retrieving a stationary potential for the wave equation with Dirichlet data and discontinuous principal coefficient from a single time-dependent Neumann boundary measurement.

Axel Osses
(CMM-DIM)

Some global Carleman weights for inverse and controllability problems

We review some recent works concerning global Carleman weights and Carleman inequalities that are adapted to (1) one measurement inverse problems for the heat and wave equations with discontinuous coefficients in the principal part, (2) null-controllability for a Navier-Stokes-rigid solid problem in variable domains and (3) locally supported boundary observations for recovering coefficients from for the wave equation.

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Bartlomiej Siudeja
Pardue University

Sharp Bounds for the Frequencies of the Vibrating Triangular Membrane

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Abstract:
A variational method is used to show new optimal bounds for the first 2 eigenvalues of the Dirichlet Laplacian on triangles (frequencies for triangular membranes). Certain other classical and recent results will also be presented.

Cesar Gomez
IMPA

On the Inverse Problem For the Risk Premium in Stochastic Volatility Models

The reconstruction of the volatility risk premium is an important element in order to calibrate stochastic volatility models using option price data from the market. In this talk, we will show how this problem comes up and then we will discuss some approaches to tackle the reconstruction problem. We will include the possibility of applying Malliavin calculus (stochastic variational calculus) to compute functional derivatives related to the reconstruction problem.

Francisco Blanco-Silva
Purdue University,

An Alternative Construction Of Curvelets. Applications To Characterization Of Regularity

Abstract. We present an alternative construction of Curvelet that simplifies both Parseval formula and Calder´on Resolution of the Identity for the Continuous Curvelet Transform. Using this transform, together with techniques of characterization of regularity developed by Holschneider and Tchamitchian, we show how to compute H¨older continuity exponents (a.k.a. Lipschitz regularity) by inspecting the asymptotic behavior of curvelet coefficients.

Fernando Guevara Vasquez
Stanford University

Electric impedance tomography with resistor networks

Electric impedance tomography consists in finding the conductivity inside a body from electrical measurements taken at its surface. This is a severely ill-posed problem: any numerical inversion scheme requires some form of regularization. We present inversion schemes that address the instability of the problem by adaptive parametrization of the

unknown conductivity. Specifically, we consider finite volume grids of size determined by the measurement precision, but where the node locations are determined as part of the problem. A finite volume discretization can be thought of as a resistor network, where the resistors are essentially averages of the conductivity over grid cells. We show that the model reduction problem of finding the smallest resistor network (of fixed topology) that can predict meaningful measurements of the Dirichlet-to-Neumann map is uniquely solvable for a broad class of measurements. To determine the unknown conductivity, we use the resistor networks to define a nonlinear mapping of the data, that behaves as an approximate inverse of the forward map. Then, we propose an efficient Newton-type iteration for finding the conductivity, using this map. We also show how to incorporate a priori information about the conductivity in the inversion scheme.

Frank Natterer
Universitat Munster

Wave equation imaging with reflectors

We consider the problem of imaging with the wave equation from reflections. It is well known that the velocity can be recovered from multi static measurements provided the source signature has zero frequencies. We show that in the presence of a plane reflector zero frequencies are not needed. The analysis is done within the Born approximation. We also show that adjoint methods work well even in the fully nonlinear case. This applies also to the case of arbitrary reflectors whose position in unknown.

Giovanni Alessandrini
University of Trieste

Open Issues Of Stability In The Inverse Conductivity Problem

It is well known that, when a-priori bounds on a _nite number of derivatives of the conductivity are given, the stability for the inverse conductivity problem is of logarithmic type, and that this rate is optimal. Still, various interesting questions remain open. In one direction, with the aim of obtaining a better stability anyhow, the following questions arise:
i) are there kinds of a-priori bounds, di_erent from regularity constraints, which are signi_cant for applications and may yield better stability results?
ii) are there functionals of the conductivity which carry relevant information on it, and depend in a stable fashion on the data? In another direction, the current interest in inverse boundary problems with local data, opens the way to the investigation of stability also in this setting. We shall discuss such issues and present some results in these directions.

Jutta Bikowski
Colorado State University

**Back To The Roots:
2D EIT Reconstructions Using Calder´On's Method**

Alberto Calder´on introduced the inverse conductivity problem by raising the question of whether the (electrical) conductivity in the interior of an object can be uniquely determined by surface measurements. Moreover he outlined a method for approximating the conductivity under the assumption that it is sufficiently close to a constant in his article 'On an inverse boundary problem'. Since then this idea found its way into several applications including electrical impedance tomography (EIT) for medical imaging and electrical resistance tomography (ERT) in geophysical science to name a few. This talk is about the pioneering work of Calderon and his method of reconstruction applied to simulated and human chest data. It is demonstrated that the original algorithm proposed by Calder´on can be used to obtain reconstructions on experimental data and holds promise as a fast algorithm for practical use.

Jennifer Mueller
Colorado State University

Cross-sectional human chest imaging by the D-bar method for electrical impedance tomography

Avid mathematical interest in the inverse conductivity problem (ICP) originated with the 1980 work "On an inverse boundary value problem" by A. Calderon. The work stimulated theoretical advances on conditions for which a unique solution exists as well as the development of algorithms for reconstructing the conductivity, in addition to the one proposed by Calderon in this seminal work. The problem has an important application in a medical imaging technique known as electrical impedance tomography (EIT) in which currents are applied on electrodes on the surface of a body, the resulting voltages are measured, and the ICP is solved to determine the conductivity distribution in the interior of the body, which is then displayed to form an image. One of numerous important theoretical works on the ICP is the global uniqueness proof by A. Nachman [Ann. of Math. 143, 1996],
in which D-bar techniques are used in a constructive proof to show that a W2,p, p¿1, conductivity is uniquely determined by the Dirichlet-to-Neumann map for a bounded 2-D Lipschitz domain. The proof relies on the existence of exponentially growing solutions to the Schrodinger equation, first used by Faddeev [Sov. Phys. Dokl. 10, 1966], but also used by Calderon in the above-mentioned work. Nachman's proof uses the D-bar method of inverse scattering, and we show that the algorithm posed by Calderon can also be viewed in the context of the D-bar method. In this talk the practical application of the algorithm outlined in Nachman's proof to human chest data for the purpose of imaging ventilation and perfusion will be discussed. Also, theoretical implications of applying this algorithm to nonsmooth conductivity distributions, as found in the human chest, will be explained, and aspectsof numerical implementation will be presented.

Jaime H. Ortega
universidad del bío-bío

Some Geometrical Inverse Problems Arising In Fluid Mechanics

in this work we consider the problem of identify an inaccessible rigid body d, which is immersed in a fluid (governed by stokes, navier-stokes or boussinesq equations) which fulfill a region . by means of boundary measurements we are interested in to obtain some geometrical information as the shape, volume or position. we present, under reasonable smoothness assumptions on and d, that one can identify the rigid body via some measurements on some part of the boundary. we also show a stability result and a numerical algorithm which allow us to recover an approximation of d. references

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Jean-Pierre Puel
Université de Versailles

Relations between some inverse problems for evolution equations and controllability results.

Some inverse problems for evolution equations (parabolic or hyperbolic) appear as dual of controllability problems. It is the case for retrieving a source or a coefficient for wave type equations, or for retrieving the state value at some time for the heat equation or linearized Navier-Stokes equations. Also the methods for solving controlability problems and for solving some inverse problems can be similar and are based on the same mathematical tools. We will analyse these facts and present some results and principles in this direction.

Jorge P. Zubelli
IMPA
(Joint work with Benoit Perthame)

On the Inverse Problem for a Size-Structured Population Model

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Kui Ren
Columbia University

Numerical Algorithm For Optical Tomography With Large Data Sets

One of the major recent advances in optical tomography is the ability to obtain large data set through the use of CCD cameras. Analytical reconstructions in simple geometry show that the use of such large data sets can potentially improve the quality of reconstruction significantly [1]. It is, however, extremely challenging to use such large data set in most model-based numerical reconstruction algorithms because of the overwhelming computational cost involved. Efforts have to be spent on designing new numerical methods to reduce the cost. We will present in this talk a numerical reconstruction procedure that allows us to use extremely large data sets while keeping the computational cost reasonable. The method is a generalization of those in [2]. Numerical examples with synthetic data will be shown to demonstrate the performance of the method.

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Lucie Baudouin
Laboratoire Analyse et Architecture Des Systemes

Inverse Problem For The Schrödinger Equation.

We would like to present an inverse problem about the Schr¨odinger equation set in a bounded domain, with time independent potential and Dirichlet boundary data. Using a Carleman estimate,
we prove the well-posedness of the inverse problem of determining the potential from measurements of the normal derivative of the solution through a part of the boundary. The detailed results and proofs are given in [1].
More precisely, we set T > 0 and let _ RN be a bounded domain and 0 be an open subset
of the C2-boundary @. We consider the Schr¨odinger equation

and the following nonlinear inverse Problem :

Is it possible to retrieve the potential q = q(x), x 2 from measurement of the normal derivative @y @_____0×(0,T)
where y is the solution to (1)?
In our study, we first prove an appropriate Carleman estimate for this Schr¨odinger equation and we then give a local answer to the questions of uniqueness and stability of this inverse problem. A geometrical condition on 0 (the same as in [4]) arises in the proof of the Carleman estimate. In the case of the wave equation, the uniqueness result for the same kind of inverse problem has been proved by M.V. Klibanov in [3] and a stability result of M. Yamamoto, deriving from it, can be read in [6]. In our proof of the stability of the inverse problem, we adapt for the Schr¨odinger equation setting an idea given by O. Yu. Imanuvilov and M. Yamamoto in [2] for the wave equation. We can also mention that the Carleman estimate proved by R. Triggiani in [5] can't be used to prove the well-posedness of our inverse problem.

Leo Tzou,
University Of Washington

Stability Estimates For The Coefficients Of The Magnetic Schr¨Odinger Equation

Abstract: In this talk we discuss a loglog-type estimate which shows that in dimension three or higher the magnetic field and the electric potential of the magnetic Schrodinger equation depends stably on the Dirichlet to Neumann (DN) map even when the boundary measurement is taken only on a subset that is slightly larger than half of the boundary. Furthermore, we prove that in the case when the measurement is taken on all of the boundary one can establish a better estimate that is of log-type. The proofs involve the use of the complex geometric optics (CGO) solutions of the magnetic Schrodinger equation constructed by G. Nakamura, Z. Sun, and G. Uhlmann. We then use these solutions as in G. Alessandrini to establish the desired stability estimate. In the partial data estimate we follow the general strategy of H. Heck and J. Wang by using a Carleman-type estimate applied to a continuous dependence result for analytic continuation developed by G. Vessella.

Matias Courdurier
University of Washington

Image Reconstruction from Limited Angle Tomographic Data With a Priori Knowledge

In Computed Tomography the goal is to recover an unknown density function f from its line integrals RL f(x)dl(x). The classic inversion formula requires information of all the line integrals in order to do this at any point. And such requirement is a very inconvenient property from the point of view of applications. We will discuss a different approach that uses a formula relating the line integrals and a Hilbert transform of the function. This way it is possible to produce uniqueness results, some stability estimates, and even inversion formulas, in cases when not all the line integrals are available.This cases are therefore not covered by the classical inversion formula. But to compensate for the incomplete knowledge of line integrals, some a priori knowledge about the support of the function, or the value of the function in some region, is needed.

Maarten V. De Hoop
Purdue University

Inverse Scattering Of Seismic Data With The Generalized Radon Transform Curvelets, Matrix Representation, Computation

We discuss the inverse scattering of seismic reflection data making use of the generalized Radon transform. Through an extension, the relevant transform attains the form of a Fourier integral operator the canonical relation of which is generated by a canonical transformation. We then introduce its associated matrix representation with respect to a frame of `curvelets'. The curvelet transform, applied to the data, is related to double beamforming used in seismic array processing. The notion of map migration appears in the relation between the inverse scattering transform, acting on a curvelet, and a pseudodi_erential operator. The matrix elements can be e_ciently computed with the aid of a separation of phase space variables (related to the notion of generalized screen expansions in seismic imaging, and a singular value decomposition). Sparsity of the matrix implies the possibility of partial reconstruction. The curvelet representation of an image, directly obtained by the matrix action on the curvelet-transformed data, can be subjected to pointwise regularity estimates for further interpretation in terms of, for example, phase transitions appearing in the mantle's
transition zone and near the core-mantle boundary. Joint research with H. Smith, G. Uhlmann (UW), F. Andersson (Lund), R.D. van der Hilst (MIT) and H. Douma (Princeton).

Masaru Ikehata
Department of Mathematics, Faculty of Engineering
Gunma University, Kiryu 376-8515, JAPAN

Virtual Signal in The Heat Equation and The Enclosure Method

Abstract
An inverse source problem and inverse initial boundary value problems for the heat equations are considered. A relationship between travel time of a virtual signal in the heat equation and exponential solutions for the backward heat equation is given. Some conjectures are formulated and discussed

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Matti Lassas
Helsinki University of Technology

Counter examples in inverse problems and invisibility

There has recently been considerable interest in the possibility, both theoretical and practical, of invisibility of objects to different types of waves. We construct several examples of cloaking enclosures covered with anisotropic materials. These examples have a close connection to earlier works carried out for the case of static Maxwell's equations (at zero frequency) [1,2], a case that is very important in electrical impedance tomography (EIT). These results have also a close connection to counter examples for inverse problem. For instance, consider the Calderón problem, that is, whether the Dirichlet to-Neumann map determines uniquely the conductivity. The problem has a positive answer in all dimensions n _ 2 if the conductivity is isotropic (under suitable regularity hypothesis) and in 2D, it also known for anisotropic conductivities. However, in all of these results it is assumed that the conductivity is bounded both below and above by strictly positive constants. If this condition is violated, one can cover any object with a properly chosen anisotropic material so that the covered object appears in all boundary measurement similar to a domain with constant conductivity. Clearly, this kind of counter example gives us theoretical instructions how to cover an object so that it appears "invisible" in zero frequency measurements. In this talk we consider similar kind of result for all frequencies. The results have been done in collaboration with Allan Greenleaf (Univ. of Rochester), Yaroslav Kurylev (Univ. of Loughborough, UK), and Gunther Uhlmann (Univ. of Washington).

Otared Kavian
Université de Versailles

Remarks on Unique Continuation Principle and Inverse Problems for Systems of Parabolic Equations

We discuss how unique continuation results for solutions of elliptic equations can be used in establishing unique continuation properties for parabolic equations, or even for a cascade system of parabolic equations. In this kind of properties the analyticity in time of the solutions plays a crucial rôle, in several aspects. We will give examples where such results are essential in the study of control problems associated to parabolic equations. We discuss also how such results can be used in the determination of coefficients (such as density and heat conduction) in parabolic equations.

Plamen Stefanov
Purdue University

Local Lens Rigidity Of A Class Of Riemannian Manifolds

Let M be a compact manifold with a Riemannian metric g on it. Then (M; g) is called lens rigid if the scattering relation, given on a subset, determines g uniquely, up to an isometry. For a class of nonsimple metrics, we show that this is true locally, near a generic set of g, including the real analytic ones. Our approach is based on the study of the linear Tensor Tomography problem of recovering a tensor eld from its integrals along a set of maximal geodesics.

Richard Melrose
Massachusetts Institute of Technology

Open problems: Potentials with radial limits

Abstract:- I will discuss work with A. Hassell and A. Vasy on scattering theory for potential perturbations of the Laplacian where the potential has radial limits at infinity and suggest possible inverse problems, particularly those related to the `Morse flow' on a compact manifold.

S. McDowall,
Western Washington University, USA

Optical tomography on simple Riemannian manifolds

Abstract
Optical tomography refers to the use of near-infrared light to determine the optical absorption and scattering properties of a medium. In the stationary Euclidean case the dynamics are modeled by the radiative transport equation, which assumes that, in the absence of interaction, particles follow straight lines. Here we shall study the problem in the presence of a (simple) Riemannian metric where particles follow the geodesic flow of the metric. This non-Euclidean geometry models a medium which has a continuously varying refractive index. We will present results for all dimensions n _ 2, but will focus on the more delicate and interesting problem in dimension two where the geometry is more apparent. We show that knowledge of the albedo operator, that which maps incoming flux to outgoing flux at the boundary, uniquely determines the absorption and scattering properties of the medium. In dimensions three and higher we assume prior knowledge of the metric while in dimension two it can be shown that the albedo operator also determines the metric.

Samuli Siltanen
Tampere University of Technology

The d-bar reconstruction method in dimension two

A direct, regularized reconstruction method for EIT is presented. The method is based on Adrian Nachman's constructive uniqueness proof for the two-dimensional inverse conductivity problem [Ann. of Math. 143, 1996]. Out of the two nonlinear steps of the proof, one is linearized and the other is not; so the resulting algorithm is nonlinear. It is shown that the reconstruction depends continuously on the data and is a smooth function. Furthermore, the algorithm is shown to give asymptotically correct results when applied the data is ideal and the regularization is weakened. Also, a connection between the d-bar method and Calderon's original linearized method is revealed.

VENKY KRISHNAN
RIEMANNIAN MANIFOLDS

A Support Theorem For Geodesic Ray Transform On Real-Analytic

We consider a compact simple Riemannian manifold M with boundary and with a real-analytic metric. Suppose we have a function f on M which satisfies the condition that the integral of f along geodesics is zero for all belonging to a certain set A . We show that if the set A satisfies a topological condition, then f = 0 on the set of points filled by these geodesics. Using this result we show that if we consider a geodesically convex subset K contained in inetrior of M and if we let A be the set of geodesics in M not intersecting K then f = 0 on the set M \ K. This provides a generalization of the classical support theorem to the Riemannian manifold setting. We use analytic microlocal analysis to prove these results. The main idea we use comes from [SU]. There Stefanov and Uhlmann use Sj¨ostrand's complex stationary phase method coupled with his FBI transform approach, [S], to study the integral geometry problem for a class of non-simple Riemannian manifolds. We combine these ideas with a theorem of Kawai-Kashiwara-H¨ormander [H, S] first used in this setting by Boman and Quinto [BQ]. We also generalize this result for the case of symmetric tensor fields. Replacing the function f above by a symmetric tensor field and under the same assumptions as above, we show that such a tensor field is potential near the boundary.

Valery Serov_
Department of Mathematical Sciences, University of Oulu,

Inverse Born approximation for two-dimensional nonlinear Schrödinger operator

This work deals with the inverse Born approximation for two-dimensional nonlinear Schr¨odinger equation with cubic nonlinearity −_u + q(x)u + _(x)|u|2u = k2u, where the real-valued unknown functions q and _ belong to Lp loc(R2) with some behaviour at the infinity. This equation has the applications in nonlinear optics. The following problem is studied: To estimate the smoothness of the terms from the Born sequence which corresponds to the scattering data with all arbitrary large energies and all angles in the scattering amplitude. These smoothness estimates allow us to conclude that the leading order singularities of the sum of unknown functions q and _ can be obtained exactly by the Born approximation. Especially, for the functions from Lp -spaces the approximation agrees with the true sum up to the functions from the Sobolev spaces. In particular, for the sum being the characteristic function of a smooth bounded domain this domain is uniquely determined by this scattering data.

Y. Kurylev, joint work with M. Lassas and A.Katsuda

Stability of boundary distance representation and reconstruction of Riemannian manifolds

A boundary distance representation of a Riemannian manifold with boundary is the set of continuous functions on the boundary, each of them being the distance function from some point inside the manifold to various points on the boundary. When this point runs over the manifold, we obtain a representation of the Riemannian manifold into the space of continuous functions on the boundary.

In this paper we study the question whether this representation determines the Riemannian manifold in a stable way if this manifold satisfies some a priori geometric bounds. The answer is affirmative, moreover, given a discrete set of approximate boundary distance functions, we construct a finite metric space that approximates the manifold in the Gromov-Hausdorff topology. In applications, the boundary distance representation appears in many in- verse problems, where measurements are made on the boundary of the object under investigation. As an example, for the heat equation with an unknown heat conductivity the boundary measurements determine the boundary distance representation of the Riemannian metric which corresponds to this conductivity.

Xiaosheng Li
Department of Mathematics
University of Washington

Identification Of Viscosity In An Incompressible Fluid

In this work we consider the unique determination of the viscosity in an incompressible fluid. Assume that a bounded domain is filled with an incompressible fluid and the velocity vector field satisfies the stationary Stokes system. We prove that the viscosity can be determined from the Cauchy data on the boundary of the domain. This is a joint work with H. Heck and J-N Wang.

We state this work more precisely. Let _ R3 be a bounded domain with smooth boundary @. Assume that is filled with an incompressible fluid. Let u = (u1, u2, u3)T be the velocity vector field satisfying the stationary Stokes system (div_μ(u, p) = 0 in divu = 0 in (1) where _μ(u, p) = 2μSym(ru) − pI and Sym(A) = (A + AT)/2 is the symmetric part of the matrix A. Here μ(x) > 0 is the viscosity function. Now let _ 2 H1/2(@) satisfy the

compatibility condition Z@ _ · nds = 0

with n being the unit outer normal of @, then there exists a unique (u, p) 2 H1()×L2() (p is unique up to a constant) solving (1) and u|@ = _. So we can define the Cauchy data of (u, p) satisfying (1) Sμ = {(u|@, _μ(u, p)n|@)} _ H1/2(@) × H−1/2(@).

In the physical sense, _μ(u, p)n|@ represents the Cauchy force acting on the boundary of the domain. The inverse problem is to determine μ from the knowledge of Sμ. We prove

the following global uniqueness result. Theorem 1. Let @ be convex with nonvanishing Gauss curvature. Assume that μ1(x) and μ2(x) are two viscosity functions satisfying μ1, μ2 2 Cn0() for n0 _ 8. Let Sμ1 and Sμ2 be the Cauchy data associated with μ1 and μ2, respectively. If Sμ1 = Sμ2 then μ1 = μ2.