

Abstracts

Alberto Corso (University of Kentucky, USA)

Title: Monomial and toric ideals associated to Ferrers graphs

Abstract: Each partition $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_n)$ determines a so-called Ferrers tableau or, equivalently, a Ferrers bipartite graph. Its edge ideal, dubbed Ferrers ideal, is a square-free monomial ideal that is generated by quadrics. We show that such an ideal has a 2-linear minimal free resolution, i.e. it defines a small sub-scheme. In fact, we prove that this property characterizes Ferrers graphs among bipartite graphs. Furthermore, using a method of Bayer and Sturmfels, we provide an explicit description of the maps in its minimal free resolution: This is obtained by associating a suitable polyhedral cell complex to the ideal/graph. Along the way, we also determine the irredundant primary decomposition of any Ferrers ideal. We conclude our analysis by studying several features of toric rings of Ferrers graphs. In particular we recover/ establish formulae for the Hilbert series, the Castelnuovo-Mumford regularity, and the multiplicity of these rings. While most of the previous works, in this highly investigated area of research, involve path-counting arguments, we offer here a new and self-contained approach based on results from Gorenstein liaison theory. (This is joint work with Uwe Nagel.)

Alicia Dickenstein (Universidad de Buenos Aires)

Title: Tropical discriminants

Abstract: Tropical geometry is used to develop a new approach to the theory of discriminants and resultants in the sense of Gelfand, Kapranov and Zelevinsky. The tropical A -discriminant, which is the tropicalization of the dual variety of the projective toric variety given by an integer matrix A , is shown to coincide with the Minkowski sum of the row space of A and of the tropicalization of the kernel of A . This leads to an explicit positive formula for the extreme monomials of any A -discriminant, and to a combinatorial rule for deciding when two regular triangulations of A correspond to the same monomial of the A -discriminant. This is a joint work with Eva Maria Feichtner and Bernd Sturmfels.

Amilcar Pacheco (UFRJ)

Title: Selmer groups of abelian varieties in extensions of function fields

Abstract: Let k be a field of characteristic q , \mathcal{C} a smooth connected curve defined over k and $A/k(\mathcal{C})$ a non-isotrivial abelian variety of dimension d with trivial $k(\mathcal{C})/k$ -trace. We suppose that $q=0$ or $q>2d+1$. For any prime number $p \neq q$, we give an upper bound for the \mathbb{Z}_p -corank of the Selmer group of $A/k(\mathcal{C})$ associated to the p -descent when we take a k -finite geometrically Galois and étale cover $\mathcal{C}' \rightarrow \mathcal{C}$. In particular, this gives a bound for the \mathbb{Z}_p -rank of the Lang-Vojta group $S_A(k(\mathcal{C}'))$.

Angelo Lopez (Roma 3)

Title: Explicit Noether-Lefschetz theorem for arbitrary threefolds.

Abstract: We study the Noether-Lefschetz locus of a very ample linear system $|L|$ on a smooth threefold Y , that is the locus of smooth surfaces S in $|L|$ such that $\text{Pic } Y \rightarrow \text{Pic } S$ is not surjective. Building on results of Green, Voisin and Otwinowska, we give explicit lower bounds, depending only on the Castelnuovo-Mumford regularity properties of L , on the codimension of the components of the Noether-Lefschetz locus of $|L|$.

Aron Simis (UFPE)

Title: The module of Kaehler differentials and associated álgebras

Abstract: This talk is based on joint work with B. Ulrich and W. Vasconcelos. I will focus on the Zariski tangent algebra and its close associate, the Rees algebra of the module of Kaehler differentials, obtained by killing torsion. There will be both geometric and arithmetic aspects considered in this exposé.

Bernd Ulrich (Purdue)

Title: Tangent álgebras

Abstract: This is a report on joint work with Aron Simis and Wolmer Vasconcelos. We study various algebraic properties of symmetric and Rees algebras of modules of differentials.

Carolina Araújo (IMPA)

Title: Rational curves and characterizations of projective varieties

Abstract: Projective varieties having rational curves through every point form an important class of varieties from the point of view of classification. These are the so called uniruled varieties. A good way of understanding the geometry of a given uniruled variety is to isolate and study a covering family of rational curves on it. This is especially efficient if the curves from this family have minimal degree with respect to some ample line bundle. Many special projective varieties, such as projective spaces, hyperquadrics, Grassmannians, etc, may be characterized by the behavior of their rational curves of minimal degree. On the other hand, consider the product of projective spaces. In this case, studying only one family of rational curves of minimal degree (the family of lines coming from one of the factors) is not enough to recover all the geometry of the variety. We must study the families of lines coming from all the factors simultaneously. In this talk I will explore some of these ideas, and give a new characterization of products of projective spaces in terms of the geometry of their families of rational curves of minimal degree.

Ciro Ciliberto (Roma 2)

Title: the Andreotti--Mayer loci and the Schottky problem.

Abstract: In this talk I will discuss some results and conjectures concerning the andreotti-mayer loci in the moduli space of principally polarized abelian varieties, with relations with the Schottky problem. This is joint work in progress with G. van der Geer.

Claudia Polini (Notre Dame)

Title: Cores of ideals

Abstract: Let I be an ideal in a commutative ring. Among the closure operations on I , the integral closure plays a central role. A reduction of I is a subideal with the same integral closure as I . One can think of reductions as simplifications of the ideal, which carry most of the information about I itself but, in general, with fewer generators. Minimal reductions, reductions minimal with respect to inclusion, are the counterpart of the integral closure. However, unlike the integral closure, minimal reductions are not unique. For this reason one considers their intersection, called the core of I . The core is related to adjoint and multiplier ideals, and to Briançon-Skoda type theorems. Furthermore a better understanding of the core could lead to a solution of Kawamata's conjecture on the non-vanishing of sections of certain

line bundles. In this talk we will discuss the importance of the core, its ubiquity in algebra and geometry, and some effective formulas for its computation.

Elham Izadi (Georgia University)

Title: The Hodge conjecture for the primitive cohomology of theta divisors.

Abstract: I will first discuss the meaning of the Hodge conjecture in general and then specialize to abelian varieties. The primitive cohomology of the theta divisor of an abelian variety gives a special Hodge structure for which one can ask whether the Hodge conjecture is true. Using Prym-embedded curves, this question was answered in the affirmative by myself and van Straten for abelian fourfolds. In this talk which is about joint work with Csilla Tamas, I will discuss the case of abelian fivefolds and show in particular that Prym-embedded curves do NOT solve the Hodge conjecture. I will, however, introduce a different family of curves which is very likely to give an answer to the Hodge conjecture.

Fernando Cukierman (UBA)

Title: Some irreducible components of spaces of foliations in projective spaces.

Abstract: Let $F_q(r, d)$ denote the space of algebraic foliations of codimension q in the complex projective space of dimension r , of degree d . We describe several new irreducible components of $F_q(r, d)$ and calculate the degree of some of them. This is a joint work with J.V. Pereira and I. Vainsencher.

Francesco Russo (UFPE)

Title: Conic(ally)-connected manifolds

Abstract: It is well known that an irreducible projective variety in P^N such that through two general points there passes a (necessarily unique) line of P^N contained in it, is a linearly embedded linear space. We shall present the ideas and the methods leading to the complete classification of smooth irreducible non-degenerate varieties in P^N such that through two general points there passes a conic contained in X (conic(ally)-connected manifolds).

Hajime Kaji (Waseda)

Title: The generic smoothness of the Gauss map and the reflexivity for a projective variety

Abstract: I discuss the relationship between the generic smoothness of the Gauss map and the reflexivity for a projective variety defined over an algebraically closed field of positive characteristic.

Israel Vainsencher (UFMG)

Title: Enumeration of hypersurfaces with exceptional tangent hyperplanes

Abstract: A plane curve has total contact with a line if they meet at a unique point. We've learned from Cicero Carvalho that these curves play a role in code theory. We give a Plücker-like formula for the number of such curves passing through the appropriate number of points. We also consider possible analogues for the case of higher dimension hypersurfaces, e.g., surfaces in P^3 passing through the appropriate number of points and possessing a tangent plane section with singularities worse than expected, say a flat point, i.e., a triple point for the corresponding tangent plane section. For instance, there are 100 cubic surfaces in a general pencil in P^3 displaying one flat point.

Jorge Vitório Pereira (IMPA)

Title: Planar Webs with Infinitesimal Automorphisms

Abstract: After recalling some classical results/concepts on web geometry I will discuss a joint work with D. Marin and L. Pirio about the structure of the space of abelian relations of planar webs admitting infinitesimal automorphisms.

Luca Chiantini (Siena)

Title: The principle of defectivity

Abstract: A projective variety X is 'defective' when some secant variety of X has dimension less than the expected value. We point out some properties of defective varieties, which seem fundamental in the applications of the theory of secant spaces. We show how some classification theorems follow from these properties, and describe the basic hierarchical structure of defective varieties.

Massimiliano Mella (Ferrara)

Title: Rational cubic hypersurfaces

Abstract: All known explicit rationality constructions for smooth cubic hypersurfaces are based on varieties with one apparent double point. This forces the dimension of the hypercubic to be even. A slight change in this approach allows to construct rational smooth hypercubic in any dimension greater or equal to 7. This leads to different perceptions of the rationality problem for cubic hypersurfaces.

Severino Collier Coutinho (UFRJ)

Title: Plane vector fields without algebraic solutions

Abstract: The existence of algebraic curves invariant under a plane vector field is related to the integrability of the corresponding first order system; which is, in its turn, related to the problem of the centre. Thus, one would like to be able to determine whether a given plane vector field has any algebraic solutions. We propose a computer algebra algorithm to do this, based on the Camacho-Sad index theorem and Noether's AF+BG theorem. This work has been done jointly with L. Menasché Schechter.

Valentina Barucci (Roma 1)

Title: Associated graded rings of one-dimensional analytically irreducible rings

Abstract: The CM-ness (CM = Cohen Macaulay) and the Hilbert series of the associated graded rings of a natural class of one-dimensional rings, including the local rings at singularities of curves, are investigated and a criterion is given. As an application, some old and new results for semigroup rings are found. The content of the talk is part of a joint paper with Ralf Fröberg, in print on Journal of Algebra.

Winnie Li (Pennsylvania)

Title: Nonlinear Codes with Improved Algebraic Geometry Bounds

Abstract: Using functions regular at certain rational points on curves over a finite field of q elements, Goppa introduced families of linear algebraic geometry codes whose information rate R plus error-correction rate δ is asymptotically bounded from below by a quantity arising from the Riemann-Roch theorem. This lower bound is called an algebraic geometry (AG) bound. When $q \geq 49$ is a square, AG bounds are known to be better than the Gilbert-Varshamov bound for δ in a certain range. By exploiting higher derivatives

of the rational functions used in Goppa's construction, Xing constructed nonlinear AG codes with $r + \delta$ better than the AG bound. This bound was improved by Elkies for square q by allowing poles as well. Niederreiter and Ozbudak extended it to all q . Stichtenoth and Xing came up with a simpler construction to obtain Elkies' bound for all q . They used Goppa's method, without exploiting higher derivatives and poles; the improvement comes from cleverly selecting functions in many Riemann-Roch spaces. The Stichtenoth and Xing's bound was improved by Niederreiter and Ozbudak, and later by Maharaj, by choosing divisors carefully and by exploiting higher derivatives. Very recently, in a joint work with Maharaj and Tsao, we improved the best known bound by exploiting the poles as well. In this survey talk, we shall explain the main idea and highlight of these constructions.

COMUNICAÇÕES

Cícero Carvalho (Uberlândia)

Title: Codes whose minimum distance beats the Goppa bound

Abstract: In this talk we would like to report on investigations concerning the construction of codes with good parameters, specially those whose minimum distance beats the so-called Goppa bound. For codes supported on one point, results of Garcia, Kim and Lax showed the importance of considering points having consecutive Weierstrass gaps. Matthews extended these results for two point codes on the Hermitian curve; her work led Homma and Kim to consider what they called "the set of pure gaps" on the Weierstrass semigroup of two points. Their work was developed by Carvalho and Torres, who generalized many of the previous results to codes supported on several points. Recently, in a joint work with T. Kato (Yamaguchi University), we showed the existence of pure gaps in the Weierstrass semigroup of points which are total inflection points on a smooth plane curve; this result together with results in our previous work allowed the construction of codes having minimum distance better than the Goppa bound, and which have better parameters than the one-point codes whose minimum distances were calculated in Yang and Kumar - On the true minimum distance of Hermitian codes, Coding Theory and Algebraic Geometry, Proceedings Luminy, 1991, Lecture Notes in Mathematics 1518 (1992), 99--107.

Cleto Brasileiro Miranda Neto (UFPE)

Title: Theory and applications of differential idealizers

Abstract: Derivations preserving (with the obvious meaning) a fixed ideal in a polynomial ring over a field give rise to a special module called *differential idealizer* (of the given ideal) geometrically, and carefully, this notion could allow us to speak about tangent vector fields leaving a given variety invariant, as well as inspire some new relation to foliation theory (about which nothing will be said). Properties and generalizations of this algebraic object will be presented, the main ones being effective criteria for its freeness and the resulting *differentially free rings*. Applications will be given specially in the context of the usual module of derivations of the above new class of rings (also, with time enough, for blowup algebras in the hypersurface case and multiplicity estimates). Finally, some questions will be raised.

Constantin Manoil (Genève)

Title: A generalisation of the dual Kummer surface

Abstract: For a curve C of genus 2 the notion of divisor in general position is defined. For such a curve, the dual Kummer surface K^* is the variety parametrizing divisors of degree 3 in general position, up to linear equivalence and antipodal involution. It is known that K^* is birationally equivalent to K , the Kummer surface belonging to the curve C . We

generalize the notion of divisor in general position for a hyperelliptic curve of genus g . Then we show that, for such a curve, there is a variety parametrizing the classes of divisors of degree $g+1$ in general position, up to antipodal involution. This is a generalization of the Kummer surface. The main result is that it is birationally equivalent to the Kummer variety belonging to the curve C . Some explicit calculus is given in genus 3.

Cristian Gonzalez (UNAB, Chile)

Title: Algebraic cycles on Severi-Brauer fibrations over curves

Abstract: In the past few years, arithmetic geometers have studied 0-cycles and codimension 2-cycles on Severi-Brauer fibrations over curves, but (unfortunately) no attention has been accorded cycles of higher dimension (or codimension) on such varieties. In this talk we present a general method for studying higher-dimensional cycles on fibered varieties. In the case of Severi-Brauer fibrations, this method should lead to finiteness theorems for the Chow groups of such varieties over certain fields.

Dan Avritzer (UFMG)

Title: Moduli space of quadratic complexes and Klein varieties

Abstract: In this talk, based on a joint work with H. Lange (University of Erlangen), I will review the theory of quadratic line complexes and associated singular surfaces which is very classical and put it in a contemporary context by considering the moduli spaces of the above objects. I will also study give the Klein varieties associated to this quadratic complexes. These are defined as follows. Let \mathcal{M} be the moduli space of (generic) quadratic complexes and \mathcal{M}_s be the moduli space of the corresponding singular Kummer surfaces. There is a surjective morphism $\pi: \mathcal{M} \rightarrow \mathcal{M}_s$. The fibres $\pi^{-1}(\pi(x))$, where x is a quadratic line complex are called the Klein variety corresponding to the quadratic complex x .

Giuseppe Borrelli (UFPE)

Title: On the bicanonical map of surfaces with a pencil of hyperelliptic curves of genus 3

Abstract: Let $f_g: S \rightarrow B$ be a surjective morphism to a smooth curve whose general fibre F is a smooth curve of genus g (then $g \geq 2$). If $g=2$, the bicanonical map of S factors through the involution induced by the g^1_2 on F . As a partial converse one has that if S is minimal, $c_1^2 \geq 10$ and φ is non-birational, then there exists an f_g with $g=2$. I will speak about surfaces admitting an f_3 with $B = \mathbb{P}^1$ and whose general fibre is a hyperelliptic curve. Namely, I will discuss the following results. Let σ be the involution induced on S . (1) If $\chi(S)=1$, then the bicanonical map of S factors through σ . In particular φ is non birational. (2) Assume S minimal. If $c_1^2 \geq 9$, then φ does not factor through σ . If $c_1^2 \geq 13$, then φ is birational.

Hemar Godinho (UnB)

Title: Anisotropic p -adic Diagonal Forms

Abstract: In the 1920's Artin conjectured that "any form of degree k in n variables has p -adic zeroes provided $n > k^2$ ". Although this result is false in general, it was proved to be true for diagonal forms by Davenport and Lewis in 1963. The form below is the classical example of a form of degree k in $n = k^2$ variables with only the trivial zero in \mathbb{Q}_p . Let $k = p-1$, $F = \sum_{j=0}^{k-1} c_j x_j^{k-j} + \dots + x_k^k$. During this seminar we would like to prove that any diagonal form of degree k in n variables, with $k^2 \geq$

$n \geq k^2 - \sqrt{\frac{k}{2}}$, without p -adic zeroes, must come from simple manipulations of the example above. For the prove we'll make use of results from additive number theory.

Ivan Pan (UFRGS)

Title: On algebraic codimension one foliations of the complex projective space with Kupka set.

Abstract: We consider algebraic codimension one foliations whose singular set is a Kupka set (i.e. the foliation looks like an open book around its singular set) of radial type. We show that such a foliation verifying some global hypotheses is a pencil.

Juliana Coelho (IMPA)

Title: Abel maps on singular curves.

Abstract: We construct Abel maps for families of curves of genus g without disconnecting nodes and show this map is a closed embedding if $g > 0$. Also, if X is a curve of genus g having disconnecting nodes, we construct an Abel map for X which is a closed embedding if X has no disconnecting lines and $g > 0$. We do not need the 1-generality condition and the result holds for any family of reduced curves (not necessarily a pencil). This is a joint work with Esteves and Caporaso.

Marco Pacini (IMPA)

Title: Spin curves over non-stable curves.

Abstract: In a recent paper, Caporaso, Casagrande and Cornalba constructed geometric meaningful compactifications of moduli spaces of theta characteristics on smooth algebraic curves, that is line bundles which are square roots of the canonical sheaf. These compactifications can be described in terms of certain line bundles on nodal curves, well known as limit square roots. In this seminar we shall consider the case of curves with cusps and tacnodes. In particular we shall give a geometric description of limits of theta characteristics when a smoothing of a curve with cusps and tacnodes is fixed. Furthermore we shall explain how to get enumerative results on such limits, using a method that can be viewed as a stable reduction for polarized curves.

Marcos Jardim (Unicamp)

Título: Stable bundles on hypersurfaces as counter-examples to a conjecture of Douglas, Reinbacher and Yau

Abstract: On a recent paper (math.AG/0604597) Douglas, Reinbacher and Yau proposed, based on string theoretical arguments, a conjecture regarding necessary conditions for the Chern classes of stable bundles on simply-connected Kähler manifolds. It amounts to a stronger version of the Bogomolov inequality. Using monads, we construct stable bundles on hypersurfaces within CP^n which are counter-examples to their conjecture.

Miriam Abdon (UFF)

Title: On certain maximal curves and Galois subcovers of the Hermitian curve

Abstract: A curve C of genus g defined over \mathbb{F}_q is called maximal if its \mathbb{F}_q -rational points attains the Hasse-Weil upper bound; i.e. $\#C(\mathbb{F}_q) = q^2 + 1 + 2qg$. The so-called Hermitian curve C_H which can be given over \mathbb{F}_q by the affine equation: $Z^q + Z = X^{q+1}$ has the biggest genus possible for maximal curves. A basic question is whether maximal curves over \mathbb{F}_q are a subcover of the Hermitian curve. Consider the curve C_{C_ℓ} defined over

\mathbb{F}_q with $q = \ell^3$ given by $y^{\ell^2} - y = x^{\ell^2 - \ell + 1}$. It was shown by Garcia and Stichtenoth that these curves are maximal and moreover for $\ell = 3$, they shown that the curve

C_3 is not a Galois subcover of the Hermitian curve. For $\ell = 2$, Garcia and Torres shown that the curve C_2 is a Galois subcover of the Hermitian curve. We can generalized the curve C_2 as follows: Consider the curve C_n defined over \mathbb{F}_q with $q = 2^n$ n an odd number $n \geq 3$, given by $y^{q^2} - y = x^{\frac{q^{n+1}}{q+1}}$. For $n = 3$ we recovered the curve C_2 . In a work in progress with L. Quoos and J. Bezerra we show that these curves

are also maximal for $n \geq 5$ and we investigate if the curve C_5 can be a Galois subcover of the Hermitian curve, but we haven't an answer yet.

Philippe Gimenez (Valladolid)

Title: Polar syzygies of algebras associated to graphs}

Abstract: Let k be a field of characteristic zero and $R = k[X_1, \dots, X_n]$ the polynomial ring in the variables X_1, \dots, X_n over k . Given a finitely generated subalgebra $A = k[f_1, \dots, f_m] \subset R$ where f_1, \dots, f_m are forms in R of the same degree, one

considers the syzygies of the transposed Jacobian module of $\{f_1, \dots, f_m\}$ and wonders when the so-called polar syzygies generate all the syzygies. When this occurs, one will say that A is polarizable. In this work, we mainly focus on algebras generated by monomials of degree two, i.e., algebras associated to a graph (with loops allowed). In this context, syzygies and polar syzygies will be described in terms of the corresponding graph and its internal combinatorics. As a consequence, one will describe well known structured classes of polarizable algebras (e.g., homogeneous coordinate ring of the 2-Veronese and Segre varieties) and relate polarizability with properties of f_1, \dots, f_m (linear syzygies) and A (normality). This is a joint work with **Isabel Bermejo** (U. La Laguna, Spain) and **Aron Simis** (U.F. Pernambuco, Brazil).

Taise Santiago (Torino)

Title: Schubert calculus on a Grassmann algebra and cohomology on the Grassmannians

Abstract: Schubert Calculus over a Grassmann Algebra" (SCGA), developed in my Ph.D. Thesis, is a certain set of formal properties common to all the *Schubert derivations* on the exterior algebra $\bigwedge M$ of an A -module M . The motivation for studying such a formalism is that it permits to deal in a unified way with different kinds of cohomologies on grassmannians such as the *classical*, the *small quantum* and the *equivariant* ones. The aim of the talk is to show the presentation in terms of generators and relations of what we call the *intersection ring* associated to a canonical \mathcal{S} -derivation (basically studied by Laksov and Thorup in a recent work) and to give, as a particular case, the presentation of the classical, small quantum and equivariant cohomology ring of the grassmannians, as well as that of the Grassmann bundles. The exposed results are joint with L. Gatto.

Teresa Monteiro Fernandes (Lisboa)

Title: Algebraic aspects of the theory of D-Modules.

Abstract: In this talk I will give a summary on the tools from commutative and non-commutative algebra which are relevant for the study of differential systems that lead, for instance, to the notion of characteristic variety, and the strict relation to phenomena that

seam to be purely geometric (the involutive property of the characteristic variety, for instance).

Tommaso de Fernex (Roma)

Title: Birational rigidity of hypersurfaces via multiplier ideals and arc spaces.

Abstract: Pukhlikov conjectured that for N greater or equal to 4, all smooth hypersurfaces of degree N in the N -dimensional projective space nonrational in a very strong sense, namely, that they are birationally superrigid, the case $N=4$ of this result being the celebrated theorem of Iskovskikh and Manin. In this talk I will give a proof of Pukhlikov's conjecture. After introducing the notion of birational rigidity and motivating it from the minimal model program, I will overview the methods (old and new) that are involved in the proof of the conjecture. This will lead me to talk about multiplier ideals, and to explain how arc spaces can be useful in order to study suitable restriction properties of these ideals.

Victor Gonzalo Lopez-Neumann (UFMG)

Title: Representation of divisor classes on hyperelliptic curves by polynomials

Abstract: For a hyperelliptic curve of genus g , a divisor in general position of degree $g+1$ is given by polynomial equations. There is an action from an algebraic group on the representations of divisors by polynomials which fixes divisor classes. This structure reduces the question of rationality of divisor classes to rationality of polynomials which is possible to control. This structure was inspired by Cassels and Flynn.

Yasuharu Amitani (Waseda University)

Title: Classification of manifolds containing low degree covers of projective space among their hyperplane sections

Abstract: Let L be a very ample line bundle on a smooth complex projective $(n+1)$ -fold X . Assume that there exists a smooth member A of $|L|$ endowed with a finite map of degree d onto projective space. The problem of classifying such pairs (X, L) was considered by several authors. In 80s, Serrano, Sommese-Van de Ven solved the problem in the case where $d=2$ and $n=1$, independently. And Fania studied the structure of (X, L) when $d=3$ and $n=1$. In 90s, Lanteri-Palleschi-Sommese considered a problem generalized with respect to dimension and degree, and solved the problem in the case where $n \geq d=2, 3$. When $n \geq d=4, 5$, Lanteri gave partial answers that contain "doubtful cases" by using the Δ -genus theory of polarized manifolds. In this talk, I will give complete classifications of (X, L) in degree 4 and 5 cases. I also explain new method for analyzing the "doubtful cases".