Abstracts

- **M. Abdon**, *Maximal curves over finite fields*
A projective geometrically irreducible algebraic curve defined over $\mathbb{F}_{q^{2}}$ is called maximal if the number of its $\mathbb{F}% _{q^{2}}$-rational points attains the Hasse-Weil upper bound% \[q+1+2gq, \] where g is the genus of the curve. In a joint work with A. Garcia, we characterize certain maximal curves by proving a weak form of a conjecture of Fuhrman, Garcia and Torres (cf. On maximal curves, J. Number Th. 67, 29-51 (1997)). For a maximal curve over $\mathbb{F}_{q^{2}}$ of genus $g=(m-1)q/2$ with m being a non-gap at some point P_{0} of the curve, they conjecture that the curve $\mathcal{X}$ is $\mathbb{F}_{q^{2}}$-isomorphic to the curve given by \[F(y)=x^{q+1}, \] where $F(y)$ is an $\mathbb{F}_{p}$-linear (or additive) polynomial of degree m (where $p=\,$char$(\mathbb{F}_{q^{2}})$). For such a maximal curve $\mathcal{X}$ we show that the point P_{0} is a rational point and, under the hypothesis that the extension $\mathbb{F}% _{q^{2}}(\mathcal{X})/\mathbb{F}_{q^{2}}(x)\;$is Galois, where div$_{\infty }(x)=mP_{0}$, we prove that the curve is $\mathbb{F}_{q^{2}}$-isomorphic to the curve given by% \[P(y)=A(x), \] where $P\;$is an additive polynomial of degree m and $A\;$is a polynomial of degree $q+1$. I\ will give several examples of non-isomorphic maximal curves satisfying the conjecture. A plane model for the later examples is of Artin-Schreier type:% \[\sum_{i=2}^{t}A_{i}z^{q/p^{i}}=x^{q+1}, \] where $q=p^{i}$, $A_{i}\in\mathbb{F}_{q^{\prime}}\subset\,\mathbb{F}_{q^{2}}$, $A_{2}=1$ and $A_{t}\neq0$. In a joint work with F. Torres, we plan to develop a necessary condition in order that such plane models be in fact $\mathbb{F}_{q^{2}}$-birational to $\mathbb{F}_{q^{2}}$-maximal curves. This necessary condition would arise as an affine variety and its computation will be a natural consequence of Weierstrass Point Theory and Frobenius orders applied to the linear series $|(q+1)P_{0}|$, where P_{0} is an $\mathbb{F}_{q^{2}}$-rational point.
- **Shreeram Abhyankar**, *Nice equations for nice groups*
I shall discuss the following two homework problems, and show how they give rise to three ways of measuring distances and angles, and eventually to the construction of nice equations with various finite classical groups as Galois groups. I shall also indicate how the raising of a polynomial to a polynomial power may lead to bringing down the ground fields of these equations from finite fields to prime fields.
HOMEWORK PROBLEM. Draw the two tangent lines to a circle C from a point P, and let L be the line joining the two points of contact. Call L the polar of P, and P the pole of L. Show that if the polar of P passes thru a point Q then the the polar of Q passes thru P.
ANOTHER HOMEWORK PROBLEM. Show that in the above problem, the circle may be replaced by any conic such as an ellipse or parabola or hyperbola, or even by a quadric or a hyperquadric.

- **Dan Avritzer**, *Curves of genus 2 and Desargues configurations*
Desargues' Theorem states that if the lines joining corresponding vertices of 2 triangles in the projective plane meet in a point then the intersections of corresponding sides lie on a line. The set of 10 points and 10 lines so abtained is called a Desargues configuration. Already in the XIX century it was found out how to relate such a configuration to a binary form of degree 6 and therefore to a curve of genus 2. The aim of this talk will be to put this set of ideas in a contemporary setting considering the moduli spaces of the objects involved. (joint work with H. Lange)

- **Rosali Brusamarello**, *Orthogonal groups containing a given maximal torus*
Let k be a field of characteristic different from 2 and let $\mathbb T$ be a fixed k-torus of dimension n. In our paper we study faithful k-representations $\rho:\mathbb T\to\mathbb{SO}(A,\sigma)$, where (A,σ) is a central simple algebra of degree $2n$ with orthogonal involution σ. Note that in this case $\rho(\mathbb T)$ is a maximal torus in $\mathbb{SO}(A,\sigma)$. We are interested in describing the pairs (A,σ) for which there is such a representation. We compute invariants for these algebras (discriminant and Clifford algebra), which are sufficient to determine their isomorphism class when $I^3(k)=0$ by a theorem of Lewis--Tignol. We will restrict our talk to the first part of the paper, which is devoted to the case where A is split over k. With

this, we will be able to give an application to a theorem of Feit on orthogonal groups over $\mathbb Q$ (field of rational numbers). This is a joint work with Jorge Morales and Pascale Koulmann.

- **M. Chardin**, *Castelnuovo-Mumford regularity and the degrees of "defining equations"*
The Castelnuovo-Mumford regularity of a projective scheme is a measure of its algebraic complexity. We will first review some known aspects of this invariant, in particular some possible definitions, and then focus on results that connects this invariant to the degrees of defining equations.

- **L. Caporaso**, *Modular properties of theta characteristics*
We show that a general, projective complex curve can be recovered from its odd theta characteristics. Or, from a different point of view, a polarized Jacobian is uniquely determined from the Gauss-images of the 2-torsion points contained in the theta-divisor. Joint work with Edoardo Sernesi.

- **L. Q. Conte**, *Curves over Finite Fields with many rational points*
The interest on curves over finite fields with many rational points (i.e., with the number N of rational places close to known upper bounds) was renewed after Goppa's construction of linear codes with good parameters from such curves. The aim of this communication is to introduce an effective method for the construction of curves over finite fields with many rational points. The method is motivated by a recent paper of van der Geer and van der Vlugt. In this method I associate a curve $\cal{X}$ over $\F_{q^n}$ to each polynomial $g(x)\in \F_{q^n}[x]$ with $\deg(g(x))\ge q^n$ and this curve $\cal{X}$ quite frequently has many rational points over $\F_{q^n}$. This is done by introducing the reduced polynomial $R(g(x))$ and then considering the curve $\cal{X}$ given by the Kummer extension of the type below: \begin{equation*} y^m=\frac{g(x)}{R(g(x))},\quad m \text{ a divisor of } (q^n-1). \end{equation*} The method is illustrate with several examples and some of the constructed curves $\cal{X}$ are really good (i.e., the number of rational points of $\cal{X}$ over the finite field in question is strictly bigger than the previously known biggest number for a curve having genus equal to the genus of this constructed curve $\cal{X}$).

- **S. Collier Coutinho**, *On the d-simplicity of polynomial rings*
Let R be a commutative ring and let d be a derivation of R. We say that R is d-simple with respect to d if $d(I) \subseteq I$ for some ideal I of R implies that $I=0$ or $I=R$. These derivations have been used to construct examples of simple rings, and also of simple modules over rings of differential operators. In this talk we discuss a method that can be applied to prove that, for certain derivations, the ring R of polynomials in two variables over the complex numbers is d-simple. The method is based on a theorem of M. Carnicer that bounds the degree of an algebraic solution of a (non-dicritical) holomorphic foliation of the complex projective plane as a function of the degree of the foliation. The method can be used both, to prove that R is d-simple with respect to a generic derivation d in a certain family, and to construct special examples of derivations with this property.

- **Luisa Doering**, *Cohomological degrees and Hilbert functions of graded modules*
This talk is based on joint work with Gunston and Vasconcelos, where we use the construction of cohomological degree function to give several estimates on the relationship between the number of generators and degrees of ideals and modules with applications to Hilbert functions. We will also present recent useful results making use of cohomological degrees given by Gunston, Rossi--Valla--Vasconcelos and Rossi--Trung--Valla.

- **Antonio Engler**, *Kaplansky's radical and a recursive description of pro-2 Galois groups*
In this talk we discuss a modified version of the ``Elementary Type Conjecture'' for pro-2 Galois groups and its connection with the Kaplansky's radical. For a field F of characteristic $\ne 2$ let $F(2)$ be its quadratic closure and denote by $\G{F}$ the corresponding Galois group. We state a condition, involving the Kaplansky's radical of F, which implies that $\G{F}$ can be obtained from some suitable closed subgroups using free pro-2 products and semi-direct group extension operations a finite number of times.

- **Marcelo Escudeiro**, *Analytic classification of curve singularities with semigroup* <6,9,19>
In this talk we will present the analytic classification of all analytic plane curves with semigroup <6,9,19> and compute all possible Tjurina invariants in this class, by means of an algorithm we

developed to make computations in the module of differentials. This is related to Heinrich's counterexample to a conjecture of Azevedo.

- **Letterio Gatto**, *Degeneration of special Weierstrass points on stable curves*
  I will report on a joint work with C. Cumino (Politecnico di Torino). Let C_0 be a stable projective complex curve of genus g. One says that a point P is limit of a special Weierstrass point on nearby smooth curves if there exists a one parameter family (X_t,P_t) of stable pointed curves ($|t|<\epsilon$) such that X_t is smooth, P_t is a special Weierstrass point for $t\neq 0$ and $P_0=P$. Suppose that C_0 is a general uninodal stable curve. Then we offer a characterization of the points which are limit of special Weierstrass points on nearby smooth curves on C_0, showing enumerative applications to the computations of certain divisor classes in the Picard Group of the Moduli Functor of stable curves of genus g.

- **Gerard van der Geer**, *Counting curves over finite fields and Siegel modular forms*
  This is a report on joint work with Carel Faber. We try to obtain information on the cohomology of the moduli space $M_{2,n}$ of n-pointed curves of genus 2 by counting the number of points of $M_{2,n}$ over finite fields. In this way we get information on the cohomology of local systems and generalized Siegel modular forms

- **Philippe Gimenez**, *On the Castelnuovo-Mumford regularity of projective monomial varieties*
  Let I be a homogeneous ideal in the polynomial ring in (n+1) variables over a field K. The Castelnuovo-Mumford regularity of I is a numerical invariant of I which provides bounds for the degrees of all its syzygies and is, in some sense, a mesure of its complexity. Its knowledge helps constructing a minimal graded free resolution of I and its computation should be easier than the determination of the syzygies -as observed in our previous joint work with Isabel Bermejo (University of La Laguna, Spain) in the case of an ideal defining a projective curve. In this work, we shall focus on the case of an ideal I defining a monomial projective variety. We shall present a combinatorial method recently developped to construct a minimal generating set and a minimal graded free resolution of I. We will show that using this method, the computation of the Castelnuovo-Mumford regularity of I is even easier than the determination of the maximal degree of an element in a minimal generating set of I. In the case of a monomial projective curve, this will apply to provide a combinatorial proof, in this case, of the known bound of Gruson-Lazarsfeld-Peskine for the regularity.

- **Dan Haran**, *Projective structure and block approximation*
  A field K is pseudo algebraically closed (PAC) if every absolutely irreducible variety defined over K has a K-rational point. The absolute Galois group of a PAC field is projective. Conversely, every projective profinite group is the absolute Galois group of some PAC field.
  There are various generalizations of this result to analogs of PAC fields and to profinite groups with families of special subgroups.
  We consider a further generalization: a field K with a family of valuations such that every absolutely irreducible variety defined over K, having rational points in the respective Henselizations has a K-rational point that is, in some sense. close to the points in the Henselizations. We characterize the absolute Galois group of such a field.

- **Abramo Hefez**, *On the classification of germs of curves*
  In this talk I will discuss the various equivalence relations among germs of analytic complex functions in two variables and the corresponding classification problem. Some particular explicit examples will be presented.

- **Hajime Kaji**, *On the reflexivity and the Gauss maps of Segre varieties*
  Abstract: In this talk, I report some results on the reflexivity and the Guass maps for Segre varieties, where a Segre variety is the image of the product of two or more projective spaces under the Segre embedding. The reflexivity of Segre varieties of two projective spaces has been studied by A. Hefez and A. Thorup: In fact, it is known that such varieties are all reflexive in any characteristic. However, this does not hold for general case. The main result here tells, for example, that the Segre variety of three projective lines is not reflexive in characteristic 2. In my talk, I will look at this Segre variety in detail. On the other hand, another result here tells that the Gauss map of a Segre variety is always an embedding. Thus, it turns out that those results yield a negative answer to an open problem raised by

S. Kleiman and R. Piene in their paper, ``On the inseparability of the Gauss map'' (Contemp. Math. 123 (1991), 107--129), as follows: \proclaim{Problem} For a projective variety $X \subseteq \Bbb P^N$ of dimension $n \ge 2$, let $\gamma : X \dashrightarrow \Bbb G(n, \Bbb P^N)$ be the Gauss map, CX the conormal variety, X^* the dual variety, and $\pi : CX \to X^*$ the natural projection. Then do γ and π have the same inseparable degree? \endproclaim

- **Gabor Korchmáros**, *Automorphism groups of maximal curves over a finite field*
Let X be a projective, geometrically irreducible, non-singular, algebraic curve defined over a finite field F of order q2 .If the number of F-rational points of X attains the Hasse--Weil upper bound, then X is called an F-maximal curve. Lachaud's theorem stating that the quotient curve of X with respect to any subgroup of the F-automorphism group Aut(X) of a F-maximal curve is still an F-maximal curve. This offers a wide possibility to derive new F-maximal curves from a known one, and hence it gives a strong motivation for the study of F-automorphism groups of an F-maximal curve X. Let G be a subgroup of Aut(X}) of a F-maximal curve. For a prime divisor p of the order of G, let H be the normal subgroup of G generated by all p-elements in G. For q even, the following classification theorem will be proved.
THEOREM
Let q be even, and let G have even order. Then one of the following cases occurs according as G fixes no or just one F-rational point of X. Either H isomorphic to one of the following groups PSU(3,22h), or SU(3,22h,), Sz(2h), h odd, SL(2,2h), or H is a Frobenius group such that the Frobenius kernel consists of all elements of odd order in H and the Frobenius complement is an abelian group with cyclic Sylow 2-group. 2)] H is a Sylow 2-group, and G is the semidirect product of H by the subgroup O(G) consisting of all elements of odd order in G. Each one of the above cases is known to occur for some values of h, apart from the possibility that H is isomorphic to SU(3,22h).

- **Herbert Lange**, *Abelian varieties with group action*
Let G be a finite group acting on a smooth projective curve X. This induces an action of G on the Jacobian JX of X and thus a decomposition of JX up to isogeny. The most prominent example of such a situation is the group G of two elements. Let X ---> Y denote the corresponding quotient map. Then JX is isogenous to the product of JY with the Prym variety of X/Y. In the talk some general results on group actions on abelian varieties are given and applied to deduce a decomposition of the jacobian JX for arbitrary group actions. Several examples are given. (Joint work with S. Recillas.)

- **Alexander Prestel**, *Representation theorems for real commutative rings*
In 1940, M.H. Stone gave an axiomatic characterization of rings C(X,R) of continuous real-valued functions on a compact space X, as partially ordered rings. This theorem gave rise to general representations of commutative rings by rings of the type C(X,R), respecting the partial ordering on C(X,R). In 1964, a representation theorem for preordered commutative rings was proved by Krivine (also known as "Kadison-Dubois" Representation Theorem). In 1999 this theorem was considerably extended by Th. Jacobi. His generalization allows interesting applications to positive polynomials and even to optimization.

- **Francesco Russo**, *On varieties with one apparent double point*
A smooth irreducible n-dimensional variety $X_n\subset\p^{2n+1}$ is said a variety with one apparent double point}, briefly VOADP, if through a general point of $\p^{2n+1}$ there passes a unique secant line to X or equivalently if a general projection into $\p^{2n}$ has a unique double point as its singularity. This condition imposes severe restrictions on the geometry of X_n. The classification of OADP-curves is trivial, the unique being the twisted cubic, while the classification of OADP-surfaces was stated by Severi and completed by the author. We will present the complete classification of OADP 3-folds and of n-dimensional OADP varieties of degree $d\leq 2n+4$. Moreover, we will illustrate how the geometrical properties of VOADP varieties cleared the path to the proof of interesting results in the classification theory of varieties (e. g. complete classification of varieties $X_n\subset\p^r$ of degree $d\leq r$) and of general properties of special kinds of varieties. We will (try to) discuss open problems and conjectures coming from classical papers on the subject.

- **Marcelo Saia**, *Deformations with constant Milnor number and multiplicity of non-degenerate complex hypersurfaces*
We investigate when a deformation of a germ of function with isolate singularity has constant Milnor

number in terms of some polyhedra associated to such germs. We consider germs that the Jacobian ideal is non-degenerate on some fixed Newton polyhedron. We show that if the germs in the family are non-degenerate on a Newton polyhedron, then the family has constant Milnor number if and only if all germs have non-decreasing Newton order with respect to the initial germ. We apply results of Greuel to show that for this kind of germs we have a positive answer for the Zariski's question: "Whether for a hypersurface singularity the multiplicity is an invariant of the topological type?"

- **Aron Simis**, *On gradient ideals and polar maps*
We will briefly survey some recent reworking on Cremona transformations in the pursue of classification results. Special situations will be mentioned so as to refer to related problems, among them a certain angle of the ideal generated by the partial derivatives of a (polynomial) form. Examples will illustrate the problems.

- **Henning Stichtenoth**, *Explicit Construction of Codes beyond the Gilbert-Varshamov Bound*
The Gilbert-Varshamov bound guarantees the existence of long linear codes with good error-correcting properties. As it was shown by Tsfasman, Vladut and Zink, there exist even better codes over finite fields of square order. However, their construction (using modular curves) is not of practical interest since the complexity of constructing these codes is very high. In this talk I give a brief survey of these results, and I will describe an algorithm for an efficient construction of codes which attain the Tsfasman-Vladut-Zink bound. The algorithm is based on an explicit tower of function fields with (asymptotically) many rational places.

- **Fernando Torres**, *On the genus of a maximal curve*
I plan to report results concerning the genus of curves that attain the Hasse-Weil bound on the number of rational points based on a joint paper with G. Korchm\'aros. The central role in our approach is Halphen's theorem applied to a natural linear series on such curves. In addition, connection with extremal Castelnuovo's curves will be discussed.

- **Israel Vainsencher**, *Genus 2, degree 5*
Let Hilb$^{5t-1}(\p3)$ be the Hilbert scheme of closed 1-dimensional subschemes of degree 5 and arithmetic genus 2 in $\p3$. Let H be the component of ${\rm Hilb}^{5t-1}(P^3)$ whose generic point corresponds to a smooth irreducible curve.
We follow the footsteps of Rojas--Vainsencher and Vainsencher--Xavier to construct an explicit desingularization of H. It is suitable for enumerative applications via Bott's residue.
The idea is to use an elementary linkage argument. Let the curve $C\subset P^3$ correspond to a general point of H. It lies on a unique quadric surface f_2. Identifying $f_2\cong P^1\times P^1$, the curve C can be seen as a curve of bidegree (2,3). Thus, it possesses ∞^1 trisecant lines, all in the system (1,0). Let L be a trisecant line. Then $L\cup C$ is a curve of bidegree (3,3). Hence, this union is a complete intersection of the quadric f_2 with a cubic surface $f_3\supset L$. We revert this construction, forming for each line L the family of intersections quadric-cubic through L. One then proceeds to identify explicit blowup centers in order to flatten the family.

- **Wolmer Vasconcelos**, *Effective Normality Criteria for Algebras of Linear Type*
The algebras studied here are subalgebras of rings of polynomials generated by 1-forms (so-called Rees algebras), with coefficients in a Noetherian ring. Given a normal domain R and a torsionfree module E with a free resolution, we study the role of the matrices of syzygies in the normality of the Rees algebra of E. When the Rees algebra of E and the symmetric algebra S(E) coincide, the main result characterizes normality in terms of an ideal of S(E) determined by the second module of syzygies of E and of the completeness of the first s symmetric powers of E (s is the rank of the first module of syzygies). It requires that R be a regular domain. Special results, under broader conditions on R, are still more effective. (Joint work with J. Brennan.)

- **Rafael Villarreal**, *Ehrhart rings and normality*
Let F be a finite set of monomials in a polynomial ring R over a field K and let P be the convex hull of the exponent vectors of the monomials in F. We will compare the Ehrhart ring of P with the integral closure (normalization) of the subring K[FT], where T is a new variable. We will dicuss the normality of various monomial subrings associated to unimodular and totally unimodular matrices

Last update: August 3, 2001