

# Normal Variance Mixture Distributions as Approximations of Poisson Mixture Sums

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By the central limit theorem and its generalizations, normal and - more generally - stable distributions turn up as weak limits of suitable scaled sums of i.i.d. random variables. When the number of summands is itself random having a Poisson mixture distribution, then normal variance mixture distributions appear as weak limits. We first relate the Kolmogorov as well as the Wasserstein distance between probability distributions to the risk measures value-at-risk and expected shortfall, respectively. We then prove upper bounds for the Wasserstein as well as for the Kolmogorov distance between distributions of Poisson mixture sums and their related normal variance mixture distributions (this corresponds to the Berry-Esseen theorem in the normal case). For this purpose, we apply size-biasing of the Poisson distribution, a conditional version of Stein's equation, and we utilize techniques established in the theory of Stein's method for the normal distribution instead of analyzing a cumbersome Stein operator for normal variance mixture distributions. It suffices that the summands are independent with equal first and second moment; the upper bounds are expressed by their third moments. Non-central limit theorems follow as a byproduct.