Optimal taxation and wealth redistribution

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We consider a society consisting of high earners and low earners, and assume that the utility derivable from income is characterized by the same function for all earners. Taxation serves two purposes: to redistribute some of the income of the high earners to the low earners, and to provide an income to government to fund its activities. In the case of a standard concave utility function, redistribution of income increases overall utility. This can be understood as the justification for redistribution. Thus, if income is redistributed, then a fraction of the total income can be redirected to the government via taxes, while still leaving an overall increase in the utility gained by earners. The maximum achievable rate of taxation is determined by the condition that the overall utility of the earners is unchanged after the redistribution of income. We assume (a) that after redistribution the high earners will earn at least what the low earners earn, and (b) that taxation is bounded by the condition that the overall utility of the earners is not diminished by the combined process of taxation and income redistribution. Some examples are worked out for specific choices of the utility function, and various generalizations are considered. It then comes as a surprise that there is an interesting analogy in the resulting mathematical framework with classical thermodynamics. The problem of optimal taxation is equivalent to that of determining the maximal amount of work that can be extracted from a thermodynamic system that is out of equilibrium. The case of earners with logarithmic utility corresponds to the situation where the problem is to take two samples of an ideal gas at two different temperatures and extract the maximal amount of useful work from the system while bringing the two samples to the same temperature.