Discrete and continuum modeling of biological network formation

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Motivated by recent papers describing rules for natural network formation in discrete settings, we propose an elliptic-parabolic system of partial differential equations. The model describes the pressure field due to Darcy's type equation and the dynamics of the conductance network under pressure force effects with a diffusion rate representing randomness in the material structure. After a short overview of the principles of discrete network modeling, we will show how to derive the corresponding macroscopic (continuum) description. The highly unusual structure of the resulting PDE system induces several interesting challenges for its mathematical analysis. We give a short overview of the tools and tricks that can be used to overcome them. In particular, we present results regarding the existence of weak solutions of the system, based on recent results on elliptic regularity theory. Moreover, we study the structure and stability properties of steady states that play a central role to understand the pattern capacity of the system. We present results of systematic numerical simulations of the system that provide further insights into the properties of the network-type solutions.