

# Functional convex order preserving approximations with application to the pricing of derivatives

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We explore the functional convex order for various classes of continuous time martingales: Brownian or Lévy driven diffusions with respect to their diffusion coefficient, stochastic integrals with respect to their integrand. Each result is bordered by counterexamples. Our approach combines the propagation of convexity results through (simulable) discrete time recursive dynamics relying on a backward dynamic programming principle and powerful functional limit theorems to transfer the results to continuous time models. In a second part, we extend this approach to optimal stopping theory, namely to the Snell envelopes of adapted functionals of (jump) martingale diffusions. Applications to various types of bounds for the pricing of pathwise dependent European and American options in local volatility models are detailed. Doing so, earlier results are retrieved in a unified way and new ones are proved. This systematic paradigm provides tractable numerical methods preserving functional convex order which may be crucial for applications, especially in Finance.