Turing instability in a model with two interacting Ising lines

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In [1], the author introduces a reaction-diffusion system to model the pattern formation phenomena present in morphogenesis. Under the assumption that the reaction part of the system is stable around an equilibrium point, he finds conditions over the diffusion coefficients under which the hole system is unstable due to the amplification of non-zero Fourier modes. This phenomena is known as Turing instability.

In this talk, we introduce an interacting particle system at which the later phenomena is present. The system is a continuous-time Markov process that has two coupled discrete toruses with Ising spins as state-space. The evolution in each torus responds to macroscopic ferromagnetic Kac's potentials, while the spins in different toruses interact in a local attractive-repulsive way. About this model, we prove hydrodynamic limit, we find conditions that guarantee the occurrence of Turing instability, we study fluctuations around its equilibrium point, and we prove that pattern formation occurs at a time that converges to the critical time at which the process starts to be finite.

[1] A. M. Turing, The chemical basis of morphogenesis.