Phase transition in the loop $O(n)$ model.

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The loop $O(n)$ model is a model for a random collection of non-intersecting loops on the hexagonal lattice, which is believed to be in the same universality class as the spin $O(n)$ model. It has been predicted by Nienhuis that for $0 \leq n \leq 2$ the loop $O(n)$ model exhibits a phase transition at a critical parameter $x_c(n) = 1/\sqrt{2+\sqrt{2-n}}$. For $0 < n \leq 2$, the transition line has been further conjectured to separate a regime with short loops when $x < x_c(n)$ from a regime with macroscopic loops when $x \geq x_c(n)$.

In this talk we will prove that for $n \in [1, 2]$ and $x = x_c(n)$ the loop $O(n)$ model exhibits macroscopic loops. A main tool in the proof is a new positive association (FKG) property shown to hold when $n \geq 1$ and $0 < x \leq \frac{1}{\sqrt{n}}$. This property implies, using techniques recently developed for the random-cluster model, the following dichotomy: either long loops are exponentially unlikely or the origin is surrounded by loops at any scale (box-crossing property). We develop a 'domain gluing' technique which allows us to employ Smirnov's parafermionic observable to rule out the first alternative when $x = x_c(n)$.