IMPA, August 1-4, 2017

Workshop on Conservation Laws and Applications

celebrating the 70th birthday of Dan Marchesin

Program & Abstracts

impa

atividade do INCTMat

auxílio

CAPES CNPq FAPERJ

Conservation Laws and Applications
celebrating the 70th birthday of Dan Marchesin

IMPA, Rio de Janeiro, August 1 – 4, 2017

The Workshop is focused on fundamental and applied topics in the theory of hyperbolic conservation laws. It is aimed for a discussion of questions that range from pure mathematical issues (existence, uniqueness and structure of solutions) to mathematical modeling and numerical analysis in fluid dynamics and problems of porous media.

Scientific Committee
Johannes Bruining (TU Delft, The Netherlands)
Alexei A. Mailybaev (IMPA, Brazil)
Aparecido J. de Souza (UFPB, Brazil)

Organizing Committee
Alexei A. Mailybaev (IMPA)
Amaury Alvarez Cruz (IMPA)

Dan Marchesin

The field of mathematics has always encompassed a broad range of ideas, from the practical to the abstract. Euclid's geometry theorems arose ultimately from the problem of measuring farmland, and Newton's calculus was motivated by questions about motion and gravity and enabled celestial predictions. Mathematics has also been inspired by detailed exploratory calculations. Gauss (when only 15 or 16) used a table of logarithms to count prime numbers and was led to conjecture the prime number theorem.

In the twentieth century, mathematics received great impetus by the increasing pace of scientific and technological development. The last half-century has seen the stupendous transformation of society by networked digital computers; mathematics has likewise been transformed, in the questions it seeks to answer, in the manner of communication and collaboration among mathematicians, and in the depth to which it is guided by computational exploration.

Dan Marchesin, who turned 70 this year, has been at the forefront of computational mathematics ever since he typed programs onto stacks of punched cards at the Courant Institute of Mathematical Sciences at New York University. (Dan relates that he learned a bitter lesson when he once dropped a program stack: number the cards!) His productive research career, which has focused on fluid dynamics, exemplifies the fruitful interplay among abstract mathematics, computational experiment, and difficult technological challenges.

Sophisticated application of mathematics to fluid dynamics dates from the time of Newton, D. Bernoulli, and Euler. A century ago, a surge in progress was sparked by the invention of airplanes and rockets. (This progress included prescient analysis of novel “finite difference” methods for simulating fluid flow, only later implemented on computers.) Many advances in fluid dynamics are founded upon the contributions of Courant, Friedrichs, Isaacson, Gelfand, Oleinik, Lax, and Glimm to the mathematical theory of conservation laws. (In essence, fluid flow is governed by the laws of conservation of mass, momentum, and energy.) The importance of this theory swelled when powerful computers became common and computational fluid dynamics became an essential tool in many areas of engineering. Dan began his scientific career at this time and has contributed notably to this growth.

Dan Marchesin was born in Bucharest, Romania, in 1947. Dan is married to Miyoko Ohtani, who has been wonderfully supportive of his scientific work, and they have one son Andrew, who is a successful engineer.

When he was very young, Dan and his family lived in Italy; after a short period spent in France, they moved to Rio de Janeiro, Brazil, in 1956. Ever since his early childhood, Dan has had an intense interest in science. This fascination motivated him to begin his college career at the Pontifical Catholic University of Rio de Janeiro (PUC-Rio) studying in the Physics Department while simultaneously studying mathematics. He graduated with a Bachelor's degree at the age of 23 and later earned his Master's degree in Mathematical Physics from the Mathematics Department at PUC-Rio.

Dan continued his studies in the doctoral program at the Courant Institute. He completed his Ph.D. in 1978 under the guidance of the outstanding mathematician James Glimm. At this time he began to study fluid dynamics using computers, which were only then becoming accessible to researchers at universities. During his post-doctoral studies at the Rockefeller University, the Courant Institute, and the Goddard Space Flight Center of NASA, Dan acquired expertise in numerical weather forecasting and in the flow of oil and brine through porous rock in petroleum reservoirs.

Dan, Miyoko, and Andrew returned to Brazil in 1980. As a young professor at PUC-Rio, Dan initiated inter-disciplinary postgraduate education in the field of Applied Mathematics. As throughout his career, he participated actively in the training of students as scientific researchers and in advancing new areas of application of mathematics.

Dan continued his scientific research at the Instituto de Matemática Pura e Aplicada (IMPA) after moving there in 1987. He founded the Laboratory of Fluid Dynamics and a program of study in this field that includes early undergraduate through post-doctoral students. While at IMPA, Dan has supervised the Applied Mathematics theses of 20 Ph.D. and 18 Master's degree students (out of a total of 46 and 53, respectively, at the Laboratory of Fluid Dynamics), and he has initiated numerous undergraduate students in scientific research with clever mathematical and computational tasks.

Among Dan Marchesin's many lasting contributions to science are the following:

- His extensive and detailed study of fundamental solutions of the equations governing three-phase immiscible flow in a porous medium. These solutions, or waves, serve as building blocks of general flows. He has elucidated the properties of such waves using an uncommon scientific approach: focusing attention not on simplified models but rather on a difficult realistic model that exhibits the full spectrum of complicated behavior.

- His discovery, in collaboration with colleagues, of the existence of a new type of wave in three-phase porous medium flow. This new wave clarifies the efficacy of modern enhanced oil recovery techniques.

- His contributions to an array of mathematical and computational specialties through work on concrete applications, from partial differential equations and dynamical system to numerical analysis and computer graphics.

- His development of mathematical and computational tools for solving and understanding fundamental solutions in fluid dynamics, ranging from the abstract (the wave manifold and bifurcation theory) to the practical (parallel computer methods for solving highly non-linear equations).

- His recent deduction of the mathematical structure of compositional models of geo-chemical flows and improved understanding of enhanced oil recovery methods based on injecting brine and carbon dioxide.

Dan has published more than one hundred papers in international scientific journals on various topics: in addition to the aforementioned results, he has reported work on computational aspects of quantum field theory, simulation and analysis of laser ionization, efficient numerical methods for weather prediction, and numerical methods for flows containing discontinuous waves.

Dan is well known for his heartfelt loyalty and friendship. We, his numerous students, colleagues, and friends, congratulate Dan on his scientific accomplishments and anticipate his continued success.

Bradley J. Plohr
Los Alamos National Laboratory

Amaury A. Cruz
IMPA

Conservation Laws and Applications

celebrating the 70 th birthday of Dan Marchesin

IMPA, Rio de Janeiro, August 1 – 4, 2017

Room 224

Hour	Tuesday, 1
08:30 - 09:00	**Check-in**
09:00 - 09:10	***Opening***
	Chair: Alexei Mailybaev
09:10 - 09:40	*Recalling the Riemann problem for the polymer injection model in a three-phase flow in a porous medium* **Aparecido J. de Souza (UFPB)**
09:40 - 10:10	*Scaling behavior of stochastic, multiphase flow in porous media* **Frederico Furtado (University of Wyoming)**
10:10-10:40	*The effect of salinity on immiscible displacement recovery* **Hans Bruining (TU Delft)**
10:40 - 11:10	**Coffee break**
	Chair: Amaury Alvarez
11:10-11:35	*Do General Relativistic Shock Wave Interactions Create Regularity Singularities in Spacetime?* **Moritz Reintjes (Instituto Superior Técnico, Lisboa)**
11:35-12:00	*On geometric issues for systems of Conservation Laws* **Pablo Castaneda (Instituto Tecnológico Autónomo de México)**

Room 224

Hour	Wednesday, 2
	Chair: Grigori Chapiro
09:10 - 09:30	*Non local solutions in conservation laws* **Vitor Matos (Universidade do Porto, Portugal)**
9:30-9:50	*Mathematical theory of geochemical injection problems for multicomponent two phase flow in porous media* **Wanderson J. Lambert (Universidade Federal de Alfenas)**
9:50-10:10	*Computing numerical solutions of the modified Buckley-Leverett equation with a dynamic nonequilibrium capillary pressure model* **Eduardo Abreu (University of Campinas, Brazil)**
10:10-10:30	*Resonant rarefaction and shock waves for a system of conservation laws* **Amaury C. Alvarez (IMPA)**
10:30 - 11:00	**Coffee break**
	Chair: Vitor Matos
11:00-11:30	*Towards automatic solvers of Riemann problems* **Dan Marchesin (IMPA)**
11:30-12:00	*Phase-field dynamic fracture model and discretization* **Marcus Sarkis (Worcester Polytechnic Institute)**
17:30	***Cocktail (Researchers room 329 at the 3rd floor, IMPA)***

Conservation Laws and Applications

celebrating the 70 th birthday of Dan Marchesin

IMPA, Rio de Janeiro, August 1 – 4, 2017

Room 224

Hour	Thursday, 3
	Chair: Eduardo Abreu
09:10 - 09:30	*Semi-continuous thermodynamics theory for a multiphase and multicomponent reservoir simulation framework* **Fabio P. Santos (Federal University of Rio de Janeiro)**
9:30-9:50	*Mathematical modeling of one-dimensional oil displacement by combined solvent-thermal flooding* **Adolfo Pires (Universidade Estadual do Norte Fluminense)**
9:50-10:10	*Traveling wave solutions describing combustion waves in porous media* **Grigori Chapiro (Universidade Federal de Juiz de Fora)**
10:10-10:30	*Non-local 1D conservation laws: when the shock wave is unstable* **Alexei A. Mailybaev (IMPA)**
10:30 - 11:00	**Coffee break**
	Chair: Frederico Furtado
11:00-11:30	*A Stable Manifold Theorem for a class of degenerate evolution equations and decay of kinetic shock layers* **Kevin Zumbrun (Indiana University)**
11:30-12:00	*Anomalous dissipation and spontaneous stochasticity in Burgers Equation* **Theodore D. Drivas (Princeton University)**

Room 224

Hour	Friday, 4
	Chair: Pablo Castaneda
09:10 - 09:30	*Numerical Analysis of Traveling Waves in Flow of Porous Media* **Ismael de Souza Ledoino (LNCC)**
9:30-9:50	*Cauchy problem for a model of combustion in a porous medium with several parallel layers* **Jesus Carlos da Mota (Universidade Federal de Goiás)**
9:50-10:10	*The effects of capillarity diffusion on non strictly hyperbolic systems of conservation laws* **Luis Fernando Lozano G.**
10:10 - 10:40	**Coffee break**
	Chair: Moritz Reintjes
10:40-11:10	*Riemann problems with delta-shock waves* **Marcelo Santos (University of Campinas, Brazil)**
11:10-11:40	*Wave manifold: topology meets PDE* **Cezar Eschenazi (Universidade Federal de Minas Gerais)**
11:40	***Closing***

Invited Speakers

Computing numerical solutions of the modified Buckley-Leverett equation with a dynamic nonequilibrium capillary pressure model

Eduardo Abreu [1], Jardel Vieira[2]

[1,2] University of Campinas - IMECC/Department of Applied Mathematics

We are interested in for solving a pseudo-parabolic partial differential equation, which models incompressible two phase flow in porous media taking into account dynamic nonequilibrium effects in the capillary pressure. We briefly discuss two numerical schemes based on the operator splitting technique. Our numerical experiments show that the standard splitting, widely used to solve parabolic problems, may fail when applied to pseudo-parabolic models. As an illustration, we give an example for this case. So we present an operator splitting scheme based on a dispersive-like character that obtains correct numerical solutions. Then, we discuss a new unsplit efficient numerical modelling, locally conservative by construction. This framework is based on a fully coupled space-time mixed hybrid finite element/volume discretization approach in order to account for the delicate local nonlinear balance between the numerical approximations of the hyperbolic flux and the pseudo-parabolic term, but linked to a natural dispersive-like character of the full pseudo-parabolic equation. We compare our numerical results with approximate solutions constructed with methods recently introduced in the specialized literature, in order to establish that we are computing the expected qualitative behaviour of the solutions.

References

[1] S. M. Hassanizadeh and W. G. Gray, *Thermodynamic basis of capillary pressure in porous media*, Water Resour. Res. 29 (1993) 3389-3405.

[2] C. van Duijn, L. Peletier and I. S. Pop *A new class of entropy solutions of the Buckley-Leverett equation*, SIAM Journal on Applied Mathematics 39 (2007) 507-536.

[3] K. Spayd and M. Shearer, *The Buckley-Leverett Equation with Dynamic Capillary Pressure* SIAM Journal on Applied Mathematics 71(4) (2011) 1088-1108.

[4] E. Abreu and J. Vieira *Computing numerical solutions of the pseudo-parabolic Buckley-Leverett equation with dynamic capillary pressure*, Mathematics and Computers in Simulation 137 (2017) 29-48.

1

[1]E. Abreu was supported by CNPq 445758/2014-7 and J. VIEIRA was supported by a CNPq fellowship.

Resonant rarefaction and shock waves for a system of conservation laws

A. C. Alvarez [1], D. Marchesin[2]

[1] IMPA

[2] IMPA

In this work, we study rarefaction and shock waves near of coincidence locus in a system of conservation laws of the form

$$\frac{\partial G(V)}{\partial t} + \frac{\partial uF(V)}{\partial x} = 0,$$

in which $V = V(x,t) : \mathbb{R}\times\mathbb{R}^+ \longrightarrow \Omega \subset \mathbb{R}^n$, $G(V) = (G_1(V), \cdots, G_{n+1}(V)) : \Omega \longrightarrow \mathbb{R}^{n+1}$, $F(V) = (F_1(V), \cdots, F_{n+1}(V)) : \Omega \longrightarrow \mathbb{R}^{n+1}$ and $u = u(x,t) : \mathbb{R} \times \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ is the Darcy speed. Small amplitude shocks and rarefactions waves arise in a mixed-type system of conservation laws in which there exist states where two characteristics coincide. We study such waves in detail when they appear near the codimension one coincidence locus with hyperbolic regions on both sides of this surface. The study is useful to construct the Riemann solution when waves are necessary for traversing the coincidence surfaces.

THE EFFECT OF SALINITY ON IMMISCIBLE DISPLACEMENT RECOVERY

Dan Solomon, Rouhi Farjzadeh and Hans Bruining[1]

[1] TU Delft, Civil Engineering and Geosciences, Stevinweg 1, 2628 CE Delft, The Netherlands. E-mail: D.S.Solomon@student.tudelft.nl, R.Farajzadeh@tudelft.nl, j.bruining@tudelft.nl,

Resumo/Abstract:
Mutually soluble solvents such as carbonated water, ether saturated water, or alcohol saturated water can be used to enhance water drive recovery of oil as the residual oil is diluted by the solvent. Here we consider the brine/ ether/ hexadecane system as an example. The procedure to find the solution of the model equations was developed by joined work of the TU-Delft and IMPA. The model equations can be straightforwardly solved numerically thanks to relevant thermodynamic data that were recently found. The solution can be interpreted with an analytic method, e.g. the method of characteristics. The analytical solution and the numerical solutions show good agreement. In the presence of salt, the solubility of the solvent in the aqueous phase can be quantified using the Setchénov coefficents, which show that the solubility in the aqueous phase decreases with increasing salt concentration. By injection of low salinity water with dissolved ether it is possible to enhance the soluble solvent process.

References

[1] H. Holldorff and H. Knapp , *Binary Vapor-Liquid-Liquid Equilibrium Of Dimethyl Ether - Water And Mutual Solubilities Of Methyl Chloride And Water*, Fluid Phase Equilibria, 44 (1988) 195-299

On geometric issues for systems of Conservation Laws

Pablo Castañeda [1]

[1] ITAM

For the past six years, I have worked with Prof. Dan Marchesin on several problems in the theory and applications of nonlinear conservation laws. As a result, some ideas have been put forth to aid the construction of solutions, both analytically and numerically. Some of these ideas, pertaining to the identification and construction of "separatrix solutions" [2], are hidden in the original work of Oleĭnik for scalar conservation laws. In [1] we have shown how to exploit Oleĭnik's E-criterion for a broader vision with a geometrical view point. In this talk we will discuss how to go beyond the scalar case using the wave curve method for systems of conservation laws and the convex hull for fractional flux functions.

[1] P. Castañeda (2016) "Oleĭnik a través del espejo" *Miscelánea Mat.* **62**: 63–80.

[2] P. Castañeda, E. Abreu, F. Furtado and D. Marchesin (2016) "On a universal structure for immiscible three-phase flow in virgin reservoirs", *Comput. Geosci.* **20**: 171–185.

TRAVELING WAVE SOLUTIONS DESCRIBING COMBUSTION WAVES IN POROUS MEDIA

Grigori Chapiro[1]

[1] Universidade Federal de Juiz de Fora, Juiz de Fora, MG 36036-900, Brazil

Resumo/Abstract:
A number of models describing injection of air into a porous medium that contains a solid fuel will be presented. In [2] the model was simplified and rigorous proof of the existence of the traveling wave solution was presented under the assumption that only co-flow combustion waves existed. The stability of such solutions was studied in [1]. The results presented in [2] were generalized by taking into account the counter-flow combustion wave in [3]. All wave sequences for the general Riemann problem solution were obtained and validated through numerical simulations.

References

[1] C.G., Furtado, L., Marchesin, D., Schecter, S., *Stability of Interacting Traveling Waves in Reaction-Convection-Diffusion Systems.*, Discrete and Continuous Dynamical Systems, v. suppl., p. 258-266, 2015.

[2] C.G., Marchesin, D., Schecter, S., *Combustion waves and Riemann solutions in light porous foam.*, Journal of Hyperbolic Differential Equations, v. 11, p. 295-328, 2014.

[3] C.G., Senos, L., *Riemann solutions for counter flow combustion in light porous foam.*, Computational and Applied Mathematics, 2017.

Anomalous Dissipation and Spontaneous Stochasticity in Burgers Equation

Theodore D. Drivas [1], Gregory Eyink[2]

[1] Princeton University, USA

[2] The Johns Hopkins University

I will discuss a novel Lagrangian approach to conservation-law anomalies in weak solutions of inviscid Burgers equation, motivated by previous work on the Kraichnan model of turbulent scalar advection. I will show that the entropy solutions of Burgers possess Markov stochastic processes of (generalized) Lagrangian trajectories backward in time for which the Burgers velocity is a backward martingale. This property guarantees dissipativity of conservation law anomalies for general convex functions of the velocity. The backward stochastic Burgers flows with these properties are not unique, however. We construct infinitely many such stochastic flows, both by a geometric construction and by the zero-noise limit of the Constantin-Iyer stochastic representation of viscous Burgers solutions. The latter proof yields the spontaneous stochasticity of Lagrangian trajectories backward in time for Burgers. We conjecture that existence of a backward stochastic flow with the velocity as martingale is an admissibility condition which selects the unique entropy solution for Burgers. Finally, I will discuss the relation of our results for Burgers with incompressible Navier-Stokes turbulence, especially Lagrangian admissibility conditions for Euler solutions and the relation between turbulent cascade directions and time-asymmetry of Lagrangian stochasticity.

Wave manifold: Topology meets PDE

Cesar S. Eschenazi [1], Carlos F. B. Palmeira[2] [1] Universidade Federal de Minas Gerais

[2] Pontifícia Universidade Católica do Rio de Janeiro

In the eighties, Dan Marchesin and PJ Paes Leme showed a weird picture to a group of topologists at PUC-Rio. This was the beginning of a road that led to the definition of wave manifold associated to a system of conservation laws, as a natural setting for rarefactions, Hugoniot curves and composites. For quadratic systems of two conservation laws, the wave manifold was constructed explicitly, stable topological configurations of rarefaction, Hugoniot and composite curves were obtained and local Riemann solutions were constructed. In order to construct non-local Riemann solutions, relevant surfaces were defined in the wave manifold, lax admissibility conditions were described and the wave manifold was divided in regions according to these conditions. We aim to give an overview of these results.

References

[1] C.F.B. Palmeira, *Line fields defined by eigenspaces of derivatives of maps from the plane to itself*, Proceedings of the VIth International Conference of Differential Geometry (Santiago de Compostela, Spain), 1988, pp.177-205.

[2] E. Isaacson, D. Marchesin, C.F.B. Palmeira and B.Plohr,*A global formalism for nonlinear waves in conservation laws*, Comm. Math. Phys., 146(1992), pp.505-552.

[3] D.Marchesin and C.F.B. Palmeira, *Topology of elementary waves for mixed-types systems of conservation laws*, Journal of Dynamics and Differential Equations, 6(3)(1994),421-440.

[4] C.S. Eschenazi and C.F.B. Palmeira, *The structure of composite rarefaction-shock foliations for quadratic systems of*

conservation laws, Matemática Contemporânea, 22(2002),113-140.

[5] A.V.Azevedo, C.S. Eschenazi, D. Marchesin and C.F.B. Palmeira,*Topological resolution of Riemann problems for pairs of conservation laws*,Quarterly of Applied Mathematics, 68(2010), 375-393.

[6] C.S. Eschenazi and C.F.B. Palmeira,*Nonlocal Lax shocks in the wave manifold for quadratic systems of two conservation laws*,(in preparation).

Scaling behavior of stochastic, multiphase flow in porous media

Frederico Furtado[1]

[1] University of Wyoming

Subsurface formations are heterogeneous at all length scales, and fine scale heterogeneities, particularly in the permeability field, can have a significant impact on large scale flow. Due to the difficulty in complete and certain characterization of these heterogeneities, stochastic representations of subsurface geologic properties have become commonplace. As a result, the flow equations have stochastic coefficients, and must also have stochastic solutions. Thus predictions of flow outcomes are inherently stochastic.

We examine multiphase flow in stochastically described heterogeneous porous media. The study centers on the interplay between nonlinearity and heterogeneity in determining fluid mixing dynamics. Monte Carlo simulations are used for a quantitative analysis of this mixing. Different flow regimes, identified by the large time scaling behavior of the mixing dynamics, are characterized. This characterization provides significant guidance for uncovering effective methods (and their limits) for the scaling-up of multiphase flow systems to scales suitable for computationally inexpensive yet accurate fluid flow simulations.

Mathematical theory of geochemical injection problems for multicomponent two phase flow in porous media

W. J. Lambert [1], A. C. Alvarez[2], D. Marchesin[3], J. Bruining[4]

[1] Universidade Federal de Alfenas

[2] IMPA

[3] IMPA

[4] TU Delft

Many two phase multicomponent models for geochemical one dimensional flow are governed by a system of equations in the variables V of the form

$$\frac{\partial G(V)}{\partial t} + \frac{\partial uF(V)}{\partial x} = 0, \tag{1}$$

in which $V = V(x,t) : \mathbb{R}\times\mathbb{R}^{+} \longrightarrow \Omega \subset \mathbb{R}^{n}$, $G(V) = (G_1(V), \cdots, G_{n+1}(V)) : \Omega \longrightarrow \mathbb{R}^{n+1}$, $F(V) = (F_1(V), \cdots, F_{n+1}(V)) : \Omega \longrightarrow \mathbb{R}^{n+1}$ and $u = u(x,t) : \mathbb{R} \times \mathbb{R}^{+} \longrightarrow \mathbb{R}^{+}$ is the Darcy speed.

In this talk we will study the system of form (1), associated to the modeling of geochemical injection problems for multicomponent two phase flow in porous media. In such a model there are composition variables y_i, for $i = 1, \cdots, n-1$, saturation variables $S = (s_w, s_o)$ and the Darcy's velocity u. In term of these variables the accumulation and flux functions G_j and F_j read as:

$$G_j = \rho_{wj} s_w + \rho_{oj} s_o \quad \text{and} \quad F_j = \rho_{wj} f_w + \rho_{oj} f_o. \quad j = 1, \cdots, n+1 \tag{2}$$

where $s_w + s_o = 1$ and $f_w + f_o = 1$, here in order to simplify the notations we use $s = s_w$, $f = f_w$, $s_o = 1 - s$ and $f_o = 1 - f$.

Here f_α, which is called the *fractional flow* of phase α, for $\alpha = w, o$, depends on $S = (s_w, s_o)$ and $y = (y_1, \cdots, y_{n-1})$ and are written as $f_\alpha = \lambda_\alpha/(\lambda_w + \lambda_o)$, for $\alpha = w$ and o, in which $\lambda_\alpha = k_\alpha/\mu_\alpha$. Functions λ_α, k_α and μ_α are called mobility, relative permeability and viscosity of phase α. The coefficients ρ_{wj} and ρ_{oj} depend on

$y = (y_1, \cdots, y_{n-1})$. The index w in ρ_{wj} indicates that these functions refers to the water phase and a similar condition for ρ_{oj} in oleic phase. Here, the physical state variables consists of the space $\Omega = \{(s, y) = [0,1] \times \mathcal{K} \subset \mathbb{R} \times \mathbb{R}^{n-1}\}$, where $\mathcal{K}$ denote the $n-1$-dimensional hypercube.

The physical model consists of $n+1$ equations for each chemical species and $n+1$ unknowns (s, y, u). The system (1) with flux (2) is written, for $j = 1, \cdots, n+1$, as:

$$\frac{\partial}{\partial t}(\rho_{wj}(y)s + \rho_{oj}(y)s_o) + \frac{\partial}{\partial x}u(\rho_{wj}(y)f + \rho_{oj}(y)f_o) = 0, \qquad (3)$$

In this talk, we present the topology of phase space Ω and we prove several results about wave curves of system (shock and rarefactions), besides results about bifurcation structures. We also prove that we can project these waves in the space of chemical variables $\mathcal{K}$ and then we obtain the solution in the complete space Ω.

Numerical Analysis of Traveling Waves in Flow of Porous Media

Ismael de Souza Ledoino [1], Bradley J. Plohr[2], Dan Marchesin[3]

[1] Laboratrio Nacional de Computao Cientfica - LNCC

[2] Los Alamos National Laboratory - LANL

[3] Instituto Nacional de Matemtica Pura e Aplicada - IMPA

Abstract:
Riemann problems play a fundamental role on the analysis of solutions to PDEs in fluid dynamics. Solving such problems numerically demands a vast amount of mathematical tools, as well as considerable computation power: travelling wave solutions require solving systems of ODEs, which are defined using points of curves generated by contour packages. Many times such tasks have to be executed iteratively respecting entropy conditions, which in turn generate complicated algorithms. Our work focuses on analyzing and exposing some of these algorithms: in particular, we focus on solving riemann problems using transitional connections.

The effects of Capillarity Diffusion on Non Strictly Hyperbolic Systems of Conservation Laws

Luis Fernando Lozano G. [1], Dan Marchesin.[1]

[1] Instituto Nacional de Matemática Pura e Aplicada

Resumo/Abstract:
The Riemann Problems for non-linear three phase flow in porous media have solutions with very rich structure, possibly because of loss of strict hyperbolicity. It is standard practice to neglect the capillary terms and consider the equations from the point of view of hyperbolic conservation laws. However, many studies consider the effects of diffusion in the solution of the Riemann Problem. In [1, 2], artificial diffusion was taken into account to prove the existence and uniqueness of certain solutions to the Riemann Problem. Many useful properties of the diffusive terms were proven in [3], using a physically correct diffusion matrix and taking full account of capillarity for flows in porous media. This physically correct approach was also used in [4, 5] to develop a computational method to simulate the solutions in heterogeneous petroleum reservoirs. The addition of capillary effects by means of non-linear diffusive terms give rise to interesting effects, such as changes of viscous admissible shocks. Our objective is to use the wave curve method to find the solution to Riemann Problems in this diffusive setting.

Acknowledgements

We thank Ismael de Souza Ledoino for his continued dedication to the numerical implementation and support of the RP2 solver.

References

[1] A. V. AZEVEDO, A. J. DE SOUZA, F. FURTADO, D. MARCHESIN, B. PLOHR , *The solution by the wave curve*

method of three-phase flow in virgin reservoirs, Transp porous Med ,2010.

[2] A. V. AZEVEDO, A. J. DE SOUZA, F. FURTADO, D. MARCHESIN , *Uniqueness of the Riemann solution for three-phase flow in a porous medium* , Society for industrial and applied mathematics,2014.

[3] A. V. AZEVEDO,D. MARCHESIN, B. PLOHR , K. ZUMBRUM ,*Capillary instability in models for three-phase flow,* Z. fr Angewandte Mathematik und Physik (ZAMP),2002.

[4] E. ABREU ,J. DOUGLAS ,F. FURTADO, D. MARCHESIN,, F. PEREIRA, ,*Three-phase immiscible displacement in heterogeneous petroleum reservoirs,* Mathematics and computers in simulation, 2006.

[5] E. ABREU *Numerical modelling of three-phase immiscible flow in heterogeneous porous media with gravitational effects,* Mathematics and Computers in Simulation, 2014.

Non-local 1D conservation laws: when the shock wave is unstable

Alexei A. Mailybaev[1]

[1] IMPA

We present spontaneously stochastic solutions for inviscid non-local conservation laws, which appear immediately after the blowup. Starting with the Burgers equation and continuing with the Sabra shell model of turbulence (as well as its continuous 1D representation), I will show how the model can be mapped into a dynamical system in renormalized coordinates and time. The renormalized system has a solution in the form of a traveling wave, which describes the universal evolution after a finite-time blowup. This wave is deterministic for the Burgers equation (corresponding to a shock), while it becomes stochastic for the Sabra model.

Towards automatic solvers of Riemann problems

Dan Marchesin [1], Eli Isaacson[2], Bradley Plohr[3], Fred Furtado[4]

[1] IMPA

[2] UWyo, in memoriam

[3] LANL and Stony Brook

[4] UWyo

Riemann problems for diffusive conservation laws are highly nonlinear and often have solutions that are complex. Complexity arises from the nonlinearity coupled with the diverse scales of transport and diffusion. It is manifested as rich bifurcation structures in the solution. Riemann problems remain the cornerstone for the organization of solutions, so that understanding them is mandatory in the mathematics and physics of conservation laws. Also, numerous modern numerical methods are based on approximations of Riemann solutions. Because these and other reasons, automatic software for finding Riemann solutions is desirable. We present our work on such software and explain how it is useful in devising optimal petroleum production strategies in reservoir engineering.

Non local solutions in conservation laws

V. Matos [1], Dan Marchesin[2]

[1] FEP, Universidade do Porto

[2] IMPA

In this talk we present some results of join work with Dan Marchesin: composite envelope, non local solution and one parameter bifurcation.

Cauchy Problem for a Model of Combustion in a Porous Medium with Several Parallel Layers

Jesus Carlos da Mota [1], Marcos Roberto Batista[2]

[1] Universidade Federal de Goiás

[2] Universidade Federal de Goiás

In this work, we discuss the existence and uniqueness of the Cauchy problem for a model of combustion in a porous medium. The medium consists of n parallel layers with distinct physical properties. For two layers the problem was solved in [2]. Here we consider $n \geq 3$. The model consists of a system of n non-linear reaction-diffusion-convection equations and n autonomous EDOs. The model includes heat transfer between two neighboring layers and a reaction rate of combustion in each layer. Considering that concentrations are known functions in all layers, we show that the Cauchy problem has a unique classical solution, deffined globally in time. The techniques used are strongly based on the iterative method of upper and lower solutions (see [6]). The main references are listed below.

References

[1] Here you put the name of the authors , *the title of your reference* , the journal's name

[2] K. N. Chueh, C. C. Conley, J. A. Smoller, *Positively invariant regions for systems of nonlinear diffusion equations*, Indiana Univ. Math. J., 26 (2) (1977), 373392.

[3] J. C. da Mota, M. M. Santos, *An application of the monotone iterative method to a combustion problem in porous media*, Nonlinear Analysis: Real World Application, 12 (2010), 1192 1201.

[4] J. C. da Mota, S. Schecter, *Combustion fronts in a porous medium with two layers, Journal of Dynamics and Differential Equations*, 18 (3) (2006), 615665.

[5] J. C. da Mota, W. Dantas and D. Marchesin, *Combustion fronts in porous media*, SIAM J. Appl. Math., 62 (2002), 21752198.

[6] C. V. Pao, *Nonlinear parabolic and elliptic equations*, Plenum Press, New York and London, (1992).

[7] D. H. Sattinger, *Monotone methods in nonlinear elliptic and parabolic boundary value problems*, Indiana Univ. Math. J., 21, (1972).

[8] J. A. Smoller, *Shock waves and reaction-diffusion equations*, Second edition. Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences], 258. Springer-Verlag, New York (1994).

Mathematical Modeling of One-dimensional Oil Displacement by Combined Solvent-Thermal Flooding

A. P. Pires [1], T. A. Marotto[2]

[1,2] Laboratório de Engenharia e Exploração de Petróleo - Universidade Estadual do Norte Fluminense

Enhanced oil recovery (EOR) is defined as a set of techniques applied to improve the recovery of hydrocarbons by the injection of materials that are not normally present in the reservoir [La89]. Most EOR methods may be classified into thermal, chemical and miscible. The chemical methods improve the sweep efficiency through the reduction of the water mobility and/or interfacial tension. Thermal methods consist of injecting a fluid (heat source), which can be steam or hot water, that causes the reduction of the oil viscosity in the reservoir, and the miscible methods are based on the injection of a solvent to decrease the capillary and interfacial forces. There are large heavy oil and bitumen deposits in many areas in the world [Zh04]. Under this scenario, it is important to develop new technologies to extract the vast amount of oil from these reservoirs. Carbonated water flooding (CWF) is an improved oil recovery technique that combines the advantages of waterflooding with carbon dioxide sequestration [HiEtAl60, Na89, Pi07, SoEtAl09, DoEtAl11]. The system of governing equations that models the injection of a hot fluid containing a solvent into an oil reservoir consists of oil, solvent and water mass balance and energy conservation. In this work we present the analytical solution for the problem of 1D oil displacement by a combined thermal-solvent EOR method.

References

[1] [DoEtAl11] Dong, Y., Dondoruk, B., Ishizawa, C., Lewis, E. J., and Kubicek, T., *An experimental investigation of carbonated water flooding*, In SPE Annual Technical Conference and Exhibition, Denver, CO (2011)

[2] [HiEtAl60] Hickok, C. W., Christensen, R. J., and Ramsay, H. J. *Progress review of the K&S carbonated waterflood project*, J. Pet. Sci. Technol., 12, 20–24 (1960).

[3] [La89] Lake, W. L., *Enhanced oil recovery, Prentice-Hall* Englewood Cliffs, NJ (1989).

[4] [Na89] Nars, T. N., and McKay, A. S *Novel oil recovery processes using caustic and carbon dioxide as dual additives in hot water*, In Petroleum Conference of The South Saskatchewan Section, Regina, CA (1989).

[5] [Pi07] Picha, M. S.: Enhanced oil recovery by hot CO2 flooding *Novel oil recovery processes using caustic and carbon dioxide as dual additives in hot water*, in SPE Middle East Oil & Gas Show and Conference, Kingdon of Bahrain, BA (2007).

[6] [SoEtAl09] Sohrabi, M., Riazi, M., Jamiolahmady, M., Ireland, S., and Brown, C. *Mechanisms of oil recovery by carbonated water injection*, In International Symposium of the Society of Core Analysts, Noordwijk, HO (2009).

[7] [Zh04] Zhao, L. *Steam alternating solvent process*, In SPE International Thermal Operations and Heavy Oil Symposium and Western Meeting, Bakersfield, CA (2004).

Do General Relativistic Shock Wave Interactions Create Regularity Singularities in Spacetime?

Moritz Reintjes [1],

[1] Instituto Superior Técnico - Lisboa, Portugal

Abstract:
It is an open question whether shock wave solutions of the Einstein Euler equations contain "regularity singularities", i.e., points where the spacetime metric would be no smoother than Lipschitz ($C^{0,1}$) in any coordinate system so that locally inertial frames fail to exist. As shown in Israel's 1966 paper, a metric $C^{0,1}$ across a *single* shock surface can be smoothed to the $C^{1,1}$ regularity sufficient for locally inertial frames to exist, by a coordinate transformation, but the method of proof fails for shock wave interactions. In the first paper addressing shock wave interactions [1, 3], we construct a coordinate transformation that smooths $C^{0,1}$ metrics to $C^{1,1}$ in spherical symmetry. Thus regularity singularities are ruled out in these basic cases, but, whether such singularities exist in more complicated shock wave solutions of the Einstein equations generated by Glimm's method remains an *open problem.*

References

[1] M. Reintjes and B. Temple, *No Regularity Singularities Exist at Points of General Relativistic Shock Wave Interaction between Shocks from Different Characteristic Families*, Proc. R. Soc. A **471**:20140834. arXiv:1610.02390

[2] M. Reintjes and B. Temple, *Shock Wave Interactions in General Relativity: The Geometry behind Metric Smoothing and the Existence of Locally Inertial Frames*, (2016), 34 pages, arXiv:1610.02390.

[3] M. Reintjes, *Spacetime is Locally Inertial at Points of General Relativistic Shock Wave Interaction between Shocks from Different Characteristic Families*, (2014), arXiv:1409.5060.

[4] M. Reintjes and B. Temple, *Regularity Singularities and the Scattering of Gravity Waves in Approximate Locally Inertial Frames*, to appear: Meth. Appl. Analysis. arXiv:1506.04074.

Obtaining Shock Solutions of the Generalized Riemann-Problem for the Buckley Leverett Equation by an Asymptotic Method

Panters Rodríguez-Bermúdez [1], Fernanda V. de Sousa[2], Baldomero Valiño Alonso[3]

[1] Universidade Federal Fluminense-UFF

[2] Programa de Pós-graduação em Modelagem Computacional em Ciência e Tecnologia-UFF.

[3] Universidad de La Habana, Cuba.

Resumo/Abstract:

In this work we apply an asymptotic method developed by Maslov to obtain shock-type solutions of the generalized Riemann Problems in the Buckley Leverett equation wich model two-phase flow in a porous medium. First, we obtain a formula for the Hugoniot-Maslov chain of this problem, i.e., an ODE system with infinite-many equations which must be satisfied by the coefficients of the Asymptotic expansion of the solution. The numerical resolution of the "truncated-chain" via a Runge-Kutta type method, give us a good approximation of the shock-wave solutions. We compare qualitatively these solutions with the solutions obtained by some of the classical FDM schemes, and also with the solutions obtained by the same asymptotic method based on a previous polynomial-approximation of the Buckley-Leverett flux function. The calculations of the Hugoniot-Maslov chain were performed within framework of Colombeau's Algebra of generalized functions. The results in this work can be considered a first step towards the future application of this asymptotic method for flow problems envolving three or more fluid-phases in a porous media.

References

[1] Maslov, V.P., Propagation of Shock Waves in Isoentropic non Viscous Gas, Itogui Naúki i Téjniki. Covremiennie Problemi Matemátiki. 8, VINITI. Moscow (1977) p. 199 (in Russian).

[2] Rodríguez-Bermúdez, P. ; Valiño-Alonso, B., Hugoniot-Maslov chains of a shock wave in conservation law with polynomial flow, Mathematische Nachrichten 280 (2007) 907-915.

[3] Danilov V.G., Omelyanov G.A., Truncation of a Chain Hugoniot-Type conditions for shock waves and its justification for the Hopf equation, Preprint ESI 502, Vienna, E.Schrodinger Inst. for Math. Phys. (1997).

[4] Dobrokhotov, S.Yu., Semenov E. S., Tirozzi B., HugoniotMaslov Chains for Singular Vortical Solutions to Quasilinear Hyperbolic Systems and Typhoon Trajectory, Journal of Mathematical Sciences, 124:5 (2004), 5209-5249.

[5] Rodríguez-Bermúdez, P. ; Valiño-Alonso, B., Asymptotic Maslov's method for shocks of conservation laws systems with quadratic flux. Applicable Analysis (electronic publication: http://dx.doi.org/10.1080/00036811.2017.1293817), 2017.

[6] Bernard, S., Meril, A., Rodríguez-Bermúdez, P., Valiño-Alonso, B., Obtaining shock solutions via Maslov's theory and Colombeau algebra for conservation laws with analytical coefficients, Novi Sad Journal of Mathematics 42 (2012) 95-116.

[7] Colombeau, J.F. Elementary Introduction to New Generalized Functions, Nort-Holland Mathematics Studies 113, 1985.

[8] Oberguggenberger, M., Multiplication of Distributions and Application to Partial Differential Equations, Pitman Research Notes in Math 259 (1992). Longman Essex.

Semi-continuous thermodynamics theory for a multiphase & multicomponent reservoir simulation framework

Fabio P. Santos [1]

[1] Department of Chemical Engineering, Federal University of Rio de Janeiro

Multiphase and multicomponent mixture flow are present in a several spectrum of applications, which goes from industrial process improvement to oil extraction. Modeling a multiphase and multicomponent flow usually involves volume-averaged procedure in N and M discrete phases and components, respectively. Unfortunately, in some applications, this modeling technique can be computational intractable, e.g., oil extraction, due to the large number of compounds needed to better represent the phenomenon of interest. In order to overcome this, most of multicomponent can be represented by a probability density function (PDF) and others some compounds can be still described by a discrete framework Jatoba2014153,Laurent2009449,Lage2007782. This approach, called semi-continuous thermodynamics (SCT) Cotterman1985, is often used to reduce the computacional cost of simulation with a large number of compounds. Likewise, this present work reformulates the conservation laws in porous media using SCT theory. The numerical validation is performed by comparing the classical discrete and the SCT approaches on 1D cases. Then, this methodology is extended to a simple 3D simulation case.
Keywords: *Compositional and reactive flows in porous media, semi-continuous thermodynamics (SCT), multiphase and multicomponent flow*

References

[1] Ronald L. Cotterman, Rainer Bender, and John M. Prausnitz, *Phase equilibria for mixtures containing very many components. Development and application of continuous thermodynamics for chemical process design.* Industrial & Engineer-

ing Chemistry Process Design and Development, 24(1):194 203, 1985.

[2] L.F.C. Jatoba, P.L.C. Lage, L.F.L.R. Silva, and F.B. Campos, *Simulation of the compressible ow with mass transfer of semi-continuous mixtures using the direct quadrature method of moments*. Computers & Chemical Engineering, 64:153 166, 2014.

[3] Paulo L.C. Lage, *The quadrature method of moments for continuous thermodynamics*. Computers & Chemical Engineering, 31(7):782 799, 2007.

[4] Claire Laurent, Grard Lavergne, and Philippe Villedieu, *Continuous thermodynamics for droplet vaporization: Comparison between gamma-pdf model and qmom*. Comptes Rendus Mcanique, 337(67):449 457, 2009.

1

[1] I am grateful to Prof Paulo Lage and D.Sc Max Akira for interesting discussions.

Riemann problems with delta-shock waves

Marcelo Santos [1], Eduardo Abreu[2], Richard De la Cruz[3]

[1] UNICAMP-State University of Campinas

[2] IMECC-UNICAMP

[3] UPTC, Tunja

We shall talk about some Riemann problems for conservation laws which solution must include delta-shock waves. In particular, we show the solution for the Suliciu system (a simplification of a model for shallow viscoelastic fluids), as constructed by Richard De la cruz. We also show some numeric experiments that indicate the validation of this solution, and possibly discuss the convergence of the numerical approximation.

Phase-field dynamic fracture model and discretization

Marcus Sarkis [1], C. J. Larsen[2], S. M. Mallikarjunaiah[3]

[1,2,3] Worcester Polytechnic Institute

In this talk we describe a finite element treatment of a variational, time-discrete model for dynamic brittle fracture. We start by providing an overview of existing dynamic fracture model that stems from Griffith's theory and based on the Ambrosio-Tortorelli interface regularization. For the temporal discretization of the wave equation of motion, we consider generalized alpha-time integration algorithm, which is implicit and unconditionally stable. To accommodate the crack irreversibility, we use a primal-dual active set strategy. To resolve the crack-path accurately, we propose a simple, robust, local mesh-refinement criterion. We show that the phase-field based variational approach and adaptive finite elements provides an efficient procedure for simulating the complex crack propagation including crack-branching.

Recalling the Riemann problem for the polymer injection model in a three-phase flow in a porous medium.

Aparecido J. de Souza [1]

[1] Universidade Federal da Paraíba

In honor of the 70 years of Prof. Dan Marchesin we recall the construction of the Riemann problem solution for the polymer injection model in a three phase flow in a porous medium carried out during the development of my doctoral thesis under his supervision in the second half of the 1980s.

A Stable Manifold Theorem for a class of degenerate evolution equations and decay of kinetic shock layers

Kevin Zumbrun [1]

[1] Indiana University, USA

We establish a Stable Manifold Theorem, with consequent exponential decay to equilibrium, for a class of degenerate evolution equations $Au' + u = D(u, u)$ with A bounded, self-adjoint, and one-to-one, but not invertible, and D a bounded, symmetric bilinear map. This is related to a number of other scenarios investigated recently for which the associated linearized ODE $Au' + u = 0$ is ill-posed with respect to the Cauchy problem. The particular case studied here pertains to the steady Boltzmann equation, yielding exponential decay of large-amplitude shock and boundary layers.