Mathematical theory of geochemical injection problems for multicomponent two phase flow in porous media

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Many two phase multicomponent models for geochemical one dimensional flow are governed by a system of equations in the variables V of the form

$$\frac{\partial G(V)}{\partial t} + \frac{\partial uF(V)}{\partial x} = 0, \tag{1}$$

in which $V = V(x,t) : \mathbb{R}\times\mathbb{R}^+ \longrightarrow \Omega \subset \mathbb{R}^n$, $G(V) = (G_1(V), \cdots, G_{n+1}(V)) : \Omega \longrightarrow \mathbb{R}^{n+1}$, $F(V) = (F_1(V), \cdots, F_{n+1}(V)) : \Omega \longrightarrow \mathbb{R}^{n+1}$ and $u = u(x,t) : \mathbb{R} \times \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ is the Darcy speed.

In this talk we will study the system of form (1), associated to the modeling of geochemical injection problems for multicomponent two phase flow in porous media. In such a model there are composition variables y_i, for $i = 1, \cdots, n-1$, saturation variables $S = (s_w, s_o)$ and the Darcy's velocity u. In term of these variables the accumulation and flux functions G_j and F_j read as:

$$G_j = \rho_{wj}s_w + \rho_{oj}s_o \quad \text{and} \quad F_j = \rho_{wj}f_w + \rho_{oj}f_o. \quad j = 1, \cdots, n+1 \tag{2}$$

where $s_w + s_o = 1$ and $f_w + f_o = 1$, here in order to simplify the notations we use $s = s_w$, $f = f_w$, $s_o = 1 - s$ and $f_o = 1 - f$.

Here f_α, which is called the *fractional flow* of phase α, for $\alpha = w, o$, depends on $S = (s_w, s_o)$ and $y = (y_1, \cdots, y_{n-1})$ and are written as $f_\alpha = \lambda_\alpha/(\lambda_w + \lambda_o)$, for $\alpha = w$ and o, in which $\lambda_\alpha = k_\alpha/\mu_\alpha$. Functions λ_α, k_α and μ_α are called mobility, relative permeability and viscosity of phase α. The coefficients ρ_{wj} and ρ_{oj} depend on

$y = (y_1, \cdots, y_{n-1})$. The index w in ρ_{wj} indicates that these functions refers to the water phase and a similar condition for ρ_{oj} in oleic phase. Here, the physical state variables consists of the space $\Omega = \{(s, y) = [0, 1] \times \mathcal{K} \subset \mathbb{R} \times \mathbb{R}^{n-1}\}$, where $\mathcal{K}$ denote the $n-1$-dimensional hypercube.

The physical model consists of $n+1$ equations for each chemical species and $n+1$ unknowns (s, y, u). The system (1) with flux (2) is written, for $j = 1, \cdots, n+1$, as:

$$\frac{\partial}{\partial t}(\rho_{wj}(y)s + \rho_{oj}(y)s_o) + \frac{\partial}{\partial x}u(\rho_{wj}(y)f + \rho_{oj}(y)f_o) = 0, \qquad (3)$$

In this talk, we present the topology of phase space Ω and we prove several results about wave curves of system (shock and rarefactions), besides results about bifurcation structures. We also prove that we can project these waves in the space of chemical variables $\mathcal{K}$ and then we obtain the solution in the complete space Ω.