Anomalous Dissipation and Spontaneous Stochasticity in Burgers Equation

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I will discuss a novel Lagrangian approach to conservation-law anomalies in weak solutions of inviscid Burgers equation, motivated by previous work on the Kraichnan model of turbulent scalar advection. I will show that the entropy solutions of Burgers possess Markov stochastic processes of (generalized) Lagrangian trajectories backward in time for which the Burgers velocity is a backward martingale. This property guarantees dissipativity of conservation law anomalies for general convex functions of the velocity. The backward stochastic Burgers flows with these properties are not unique, however. We construct infinitely many such stochastic flows, both by a geometric construction and by the zero-noise limit of the Constantin-Iyer stochastic representation of viscous Burgers solutions. The latter proof yields the spontaneous stochasticity of Lagrangian trajectories backward in time for Burgers. We conjecture that existence of a backward stochastic flow with the velocity as martingale is an admissibility condition which selects the unique entropy solution for Burgers. Finally, I will discuss the relation of our results for Burgers with incompressible Navier-Stokes turbulence, especially Lagrangian admissibility conditions for Euler solutions and the relation between turbulent cascade directions and time-asymmetry of Lagrangian stochasticity.