

# The effects of Capillarity Diffusion on Non Strictly Hyperbolic Systems of Conservation Laws

Luis Fernando Lozano G. <sup>1</sup>, Dan Marchesin.<sup>1</sup>

<sup>1</sup> Instituto Nacional de Matemática Pura e Aplicada

## Resumo/Abstract:

The Riemann Problems for non-linear three phase flow in porous media have solutions with very rich structure, possibly because of loss of strict hyperbolicity. It is standard practice to neglect the capillary terms and consider the equations from the point of view of hyperbolic conservation laws. However, many studies consider the effects of diffusion in the solution of the Riemann Problem. In [1, 2], artificial diffusion was taken into account to prove the existence and uniqueness of certain solutions to the Riemann Problem. Many useful properties of the diffusive terms were proven in [3], using a physically correct diffusion matrix and taking full account of capillarity for flows in porous media. This physically correct approach was also used in [4, 5] to develop a computational method to simulate the solutions in heterogeneous petroleum reservoirs. The addition of capillary effects by means of non-linear diffusive terms give rise to interesting effects, such as changes of viscous admissible shocks. Our objective is to use the wave curve method to find the solution to Riemann Problems in this diffusive setting.

## Acknowledgements

We thank Ismael de Souza Ledoino for his continued dedication to the numerical implementation and support of the RP2 solver.

## References

- [1] A. V. AZEVEDO, A. J. DE SOUZA, F. FURTADO, D. MARCHESIN, B. PLOHR , *The solution by the wave curve*

*method of three-phase flow in virgin reservoirs*, Transp porous Med ,2010.

- [2] A. V. AZEVEDO, A. J. DE SOUZA, F. FURTADO, D. MARCHESIN , *Uniqueness of the Riemann solution for three-phase flow in a porous medium* , Society for industrial and applied mathematics,2014.
- [3] A. V. AZEVEDO,D. MARCHESIN, B. PLOHR , K. ZUMBRUM , *Capillary instability in models for three-phase flow*, Z. fr Angewandte Mathematik und Physik (ZAMP),2002.
- [4] E. ABREU ,J. DOUGLAS ,F. FURTADO, D. MARCHESIN,, F. PEREIRA, ,*Three-phase immiscible displacement in heterogeneous petroleum reservoirs*, Mathematics and computers in simulation, 2006.
- [5] E. ABREU *Numerical modelling of three-phase immiscible flow in heterogeneous porous media with gravitational effects*, Mathematics and Computers in Simulation, 2014.