

Computing numerical solutions of the modified Buckley-Leverett equation with a dynamic nonequilibrium capillary pressure model

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We are interested in solving a pseudo-parabolic partial differential equation, which models incompressible two phase flow in porous media taking into account dynamic nonequilibrium effects in the capillary pressure. We briefly discuss two numerical schemes based on the operator splitting technique. Our numerical experiments show that the standard splitting, widely used to solve parabolic problems, may fail when applied to pseudo-parabolic models. As an illustration, we give an example for this case. So we present an operator splitting scheme based on a dispersive-like character that obtains correct numerical solutions. Then, we discuss a new unsplit efficient numerical modelling, locally conservative by construction. This framework is based on a fully coupled space-time mixed hybrid finite element/volume discretization approach in order to account for the delicate local nonlinear balance between the numerical approximations of the hyperbolic flux and the pseudo-parabolic term, but linked to a natural dispersive-like character of the full pseudo-parabolic equation. We compare our numerical results with approximate solutions constructed with methods recently introduced in the specialized literature, in order to establish that we are computing the expected qualitative behaviour of the solutions.

References

- [1] S. M. HASSANIZADEH AND W. G. GRAY, *Thermodynamic basis of capillary pressure in porous media*, Water Resour. Res. 29 (1993) 3389-3405.
- [2] C. VAN DUIJN, L. PELETIER AND I. S. POP *A new class of entropy solutions of the Buckley-Leverett equation*, SIAM Journal on Applied Mathematics 39 (2007) 507-536.

- [3] K. SPAYD AND M. SHEARER, *The Buckley-Leverett Equation with Dynamic Capillary Pressure* SIAM Journal on Applied Mathematics 71(4) (2011) 1088-1108.
- [4] E. ABREU AND J. VIEIRA *Computing numerical solutions of the pseudo-parabolic Buckley-Leverett equation with dynamic capillary pressure*, Mathematics and Computers in Simulation 137 (2017) 29-48.

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