Resonant rarefaction and shock waves for a system of conservation laws

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In this work, we study rarefaction and shock waves near of coincidence locus in a system of conservation laws of the form

$$\frac{\partial G(V)}{\partial t} + \frac{\partial uF(V)}{\partial x} = 0,$$

in which $V = V(x,t) : \mathbb{R}\times\mathbb{R}^+ \longrightarrow \Omega \subset \mathbb{R}^n$, $G(V) = (G_1(V), \cdots, G_{n+1}(V)) : \Omega \longrightarrow \mathbb{R}^{n+1}$, $F(V) = (F_1(V), \cdots, F_{n+1}(V)) : \Omega \longrightarrow \mathbb{R}^{n+1}$ and $u = u(x,t) : \mathbb{R} \times \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ is the Darcy speed. Small amplitude shocks and rarefactions waves arise in a mixed-type system of conservation laws in which there exist states where two characteristics coincide. We study such waves in detail when they appear near the codimension one coincidence locus with hyperbolic regions on both sides of this surface. The study is useful to construct the Riemann solution when waves are necessary for traversing the coincidence surfaces.