The moduli space of isometric surfaces with the same mean curvature in 4-dimensional space forms

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Resumo/Abstract:
We study the moduli space of congruence classes of isometric surfaces with the same mean curvature in 4-dimensional space forms. Having the same mean curvature means that there exists a parallel vector bundle isometry between the normal bundles that preserves the mean curvature vector fields. We prove that if both Gauss lifts of a compact surface to the twistor bundle are not vertically harmonic, then there exist at most three nontrivial congruence classes. We show that surfaces with a vertically harmonic Gauss lift, allow locally a one-parameter family of isometric deformations with the same mean curvature. This family is trivial only if the surface is superconformal. For such compact surfaces with non-parallel mean curvature, we prove that the moduli space is the disjoint union of two sets, each one being either finite, or a circle. In particular, for surfaces in $\mathbb{R}^4$ we prove that the moduli space is a finite set, under a condition on the Euler numbers of the tangent and normal bundles.