Abstract for the Posters Session

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Title: Biconservative submanifolds in $\mathbb{S}^n \times \mathbb{R}$ and $\mathbb{H}^n \times \mathbb{R}$
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1 Introduction

Roughly speaking, *biconservative* submanifolds arise as the vanishing of the stress-energy tensor associated to the variational problem of biharmonic submanifolds. More precisely, an isometric immersion $f : M \to N$ between two Riemannian manifolds is biconservative if the tangent component of its bitension field is identically zero.

Simplest examples of biconservative hypersurfaces in space forms are those that have constant mean curvature. In this case, the condition of biconservative becomes $2A(\operatorname{grad} H) + H \operatorname{grad} H = 0$, where A is the shape operator and H is the mean curvature function of the hypersurface. The case of surfaces in $\mathbb{R}^3$ was considered by Hasanis-Vlachos [8], and surfaces in $\mathbb{S}^3$ and $\mathbb{H}^3$ was studied by Caddeo-Montaldo-Oniciuc-Piu [2]. In the Euclidean space $\mathbb{R}^3$, these surfaces are rotational. Recent results in the study of biconservative submanifolds were obtained, for example, in [5], [6], [7], [17], [18], [20], [21].

Apart from space forms, however, there are few Riemannian manifolds for which biconservative submanifolds are classified. Recently, this was considered for surfaces with parallel mean curvature vector field in $\mathbb{S}^n \times \mathbb{R}$ and $\mathbb{H}^n \times \mathbb{R}$ in [4], where they found explicit parametrizations for such submanifolds.

In [15], we give a complete classification of biconservative submanifolds in $\mathbb{Q}^4_\epsilon \times \mathbb{R}$ with nonzero parallel mean curvature vector field and codimension 2. This extend the one obtained in [4]. To state our result, let $\mathbb{Q}^n_\epsilon$ denote either the unit sphere $\mathbb{S}^n$ or the hyperbolic space $\mathbb{H}^n$, according as $\epsilon = 1$ or $\epsilon = -1$, respectively. Given an isometric immersion $f : M^m \to \mathbb{Q}^n_\epsilon \times \mathbb{R}$, let ∂_t be a unit vector field tangent to the second factor. Then, a tangent vector field T on M^m and a normal vector field η along f are defined by

$$\partial_t = f_* T + \eta. \tag{1.1}$$

Consider now an oriented minimal surface $\phi : M^2 \to \mathbb{Q}^2_a \times \mathbb{R}$ such that the vector field T defined by (1.1) is nowhere vanishing, where $a \neq 0$ and $|a| < 1$. Let $b > 0$ be a real number such that $a^2 + b^2 = 1$. Let now

$$f : M^3 := M^2 \times I \to \mathbb{Q}^4_\epsilon \times \mathbb{R}$$

be given by

$$f(p, s) = \left(b \cos \frac{s}{b}, b \sin \frac{s}{b}, \phi(p) \right). \tag{1.2}$$

Theorem 1.1. *The map f defines, at regular points, an isometric immersion with $\langle H, \eta \rangle = 0$, where H is the mean curvature vector field of f. Moreover, f is a biconservative isometric immersion with parallel mean curvature vector field if and only if ϕ is a vertical cylinder. Conversely, any biconservative isometric immersion $f : M^3 \to \mathbb{Q}^4_\epsilon \times \mathbb{R}$ with nonzero parallel mean curvature vector field, such that the vector field T defined by* (1.1) *is nowhere vanishing, is locally given in this way.*

In particular, we prove that the submanifolds of Theorem 1.1 belong to a special class, which consists of isometric immersions $f : M^m \to \mathbb{Q}^n_\epsilon \times \mathbb{R}$ with the property that the vector field T is an eigenvector of all shape operators of f.

The goal of this poster is to present some general results about n-dimensional biconservative submanifolds in $\mathbb{Q}^n_\epsilon \times \mathbb{R}$. Moreover, we will give the main ideas of the proof of the main Theorem in [15].

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