LOCAL HOMOLOGY WITH RESPECT TO A PAIR OF IDEALS AND SOME APPLICATIONS

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Throughout this summary, R is a commutative ring with non-zero identity. For an R-module M and an ideal I of R there are two important functors in commutative algebra and algebraic geometry which are the I-torsion functor $\Gamma_I(\bullet)$ and the I-adic completion functor $\Lambda_I(\bullet)$ defined by:

$$\Gamma_I(M) = \bigcup_{t>0} \left(0 :_M I^t\right) \text{ and } \Lambda_I(M) = \varprojlim_{t\in\mathbb{N}} M/I^t M.$$

It should be noted that the I-torsion functor $\Gamma_I(\bullet)$ is left exact and its i-th right derived functor $\mathrm{H}^i_I(\bullet)$ is called the i-th local cohomology functor with respect to I. However, the I-adic completion functor $\Lambda_I(\bullet)$ is neither right nor left exact, so computing its left derived functors is in general difficult.

The local cohomology theory of Grothendieck has proved to be an important tool in algebraic geometry, commutative algebra and algebraic topology, and makes it possible for us to obtain applications for this local cohomology module. Its dual theory of local homology is also studied by many mathematicians: Greenlees and May [5], Tarrío [1], and Cuong and Nam [2], etc. In [3], we have that Grothendieck introduced the definition of local cohomology module. Let I be an ideal of R and let M be an R-module, then the module

$$\mathrm{H}^i_I(M) = \varinjlim_{t\in\mathbb{N}} \mathrm{Ext}^i_R\left(R/I^t, M\right)$$

is called the i-th local cohomology module of M with respect to I.

In this work, we introduce the local homology module with respect to a pair of ideals (I, J) which is in some sense dual to [7] local cohomology defined by a pair of ideals. We have studied local homology module defined by a pair of ideals for linearly compact modules. It should be mentioned that the class of linearly compact modules is great, it contains important classes of modules in algebra. For example, Artinian modules are linearly compact [4, Theorem 2.1]. Moreover, if R is a complete Noetherian local ring and M is a finitely generated R-module, then M is semidiscrete (that means every submodule of M is closed) and linearly compact [6, 7.3].

We remember some definitions, results obtained for the functor of local homology $\Lambda_{I,J}(\bullet)$ defined by a pair of ideals (I, J) and to prove results for the local homology module $\mathrm{H}^{I,J}_i(M)$ defined by a pair of ideals when M is a linearly compact R-module. We put some results on vanishing of local homology modules with respect to a pair of ideals and its relation with the co-localization of a module.

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