

Asymptotics for the Late Arrivals Problem ¹

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We study a discrete time queueing system where deterministic arrivals have i.i.d. exponential delays ξ_i . The standard deviation σ of the delay is finite, but much larger than the deterministic unit interarrival time. We describe the model as a bivariate Markov chain, we prove that it is ergodic and then we focus on the unique joint equilibrium distribution. We write a functional equation for the bivariate generating function, finding the solution of such equation on a subset of its set of definition. This solution allows us to prove that the equilibrium distribution of the Markov chain decays super-exponentially fast in the quarter plane. Finally, exploiting the latter result, we discuss the numerical computation of the stationary distribution, showing the effectiveness of a simple approximation scheme in a wide region of the parameters. The model, motivated by air and railway traffic, was proposed many decades ago by Kendall with the name of "late arrivals problem", but no solution has been found so far.

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