# The divergence between two measures in product spaces

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Let $\mu$ and $\nu$ be two probability measures over the same measurable space $(\Omega, \mathcal{F})$. It is natural to ask if this two measures are related in any sense, and if there is some function to scale this relation. There are in the literature several quantities devoted to answer somehow these questions. We highlight here the *mutual information* and the *relative entropy* (or *Kullback-Leibler divergence*), which was been extensively discussed in the literature.

This work is dedicated to a quantity wich describes the degree of similarity between two given measures. As far as we know, it was never considered in the literature. Its definition will be given as follows. Let $\chi$ an alphabet. Let $\mu$ and $\nu$ two measures over $(\Omega, \mathcal{F})$, with $\Omega = \chi^{\infty}$. The $k-$divergence between $\mu$ and $\nu$ is defined by:

$$E_{\mu,\nu}(k) = \sum_{\omega \in \chi^k} \mu\nu(\omega) \ ,$$

In many cases this quantity is an exponentially decreasing sequence on $k$ and this leads to consider its limiting rate. The main result establishes conditions for the existence of the limiting rate function. We also show that in particular, when the two measures coincide, this limit corresponds to the Rényi entropy of the measure at argument $\beta = 2$.