

Quasi-additive estimates on the Hamiltonian for the One-dimensional Long Range Ising Model.*

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In this work, we study the problem of getting quasi-additive bounds for the Hamiltonian of the long range Ising model, when the two body interaction term decays proportionally to $\frac{1}{d^{2-\alpha}}$, $\alpha \in (0, 1)$. We revisit the paper by Cassandro, Ferrari, Merola & Presutti [2], where they extend to the case $\alpha \in [0, \frac{\ln 3}{\ln 2} - 1)$ the result of the existence of a phase transition by using a Peierls argument given by Fröhlich & Spencer [3] for $\alpha = 0$. The main arguments of [2] are based in a quasi-additive decomposition of the Hamiltonian in terms of hierarchical structures called Triangles and Contours, which are related to the original definition of Contours introduced by Fröhlich & Spencer in [3]. In this work, we study the existence of a quasi-additive decomposition of the Hamiltonian in terms of the Contours defined in [2]. The most relevant results are the existence of a quasi additive decomposition for the Hamiltonian in terms of Contours when $\alpha \in [0, 1)$ but not in terms of triangles. The fact that cannot be a quasi additive bound in term of triangles lead to a very interesting maximization problem whose maximizer is related to a discrete Cantor set.

As a consequence of the quasi additive bounds we prove that we can generalise the [2] result, that is a Peierls argument, to the whole interval $\alpha \in [0, 1)$. We also state here the result of [1] about cluster expansions that implies that Theorem 2.4 that concerns interfaces and Theorem 2.5 that concerns n point truncated correlation functions in [1] are valid for all $\alpha \in [0, 1)$ instead of only $\alpha \in [0, \frac{\ln 3}{\ln 2} - 1)$.

Selected References

- [1] CASSANDRO, MARZIO AND MEROLA, IMMACOLATA AND

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- [2] CASSANDRO, MARZIO AND FERRARI, PABLO AUGUSTO AND MEROLA, IMMACOLATA AND PRESUTTI, ERICO, Geometry of contours and Peierls estimates in $d=1$ Ising models with long range interactions. J. Math. Phys. (2014), 46 (5):053305.
- [3] FRÖHLICH, JÜRGEN AND SPENCER, THOMAS, Journal = The phase transition in the one-dimensional Ising model with $1/r^2$ interaction energy. Comm. Math. Phys.(1982), Volume 84, Number 1, 87-101