Minimal hypersurfaces, isometric embeddings, and manifolds with nonnegative scalar curvature

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Compact manifolds with nonnegative scalar curvature with boundary is a basic subject of study in mathematical relativity. If the boundary of the manifold is isometric to a strictly convex hypersurface in the Euclidean space, a fundamental result of Shi and Tam asserts that the total mean curvature of the boundary is no greater than the total mean curvature of the corresponding Euclidean hypersurface. In 3-dimension, Shi-Tam's result is equivalent to the Riemannian positive mass theorem.

In this talk, we present a supplement to Shi-Tam's theorem by including the boundary effect of minimal hypersurfaces. More precisely, we consider a compact manifold with nonnegative scalar curvature, whose boundary consists of two parts, the outer boundary and the horizon boundary. Here the horizon boundary is the union of all closed minimal hypersurfaces in the manifold and the outer boundary is assumed to be a topological sphere. By assuming the outer boundary is isometric to a suitable 2-convex hypersurface in a Schwarzschild manifold of positive mass m, we establish an inequality relating m, the area of the horizon boundary, and two weighted total mean curvatures of the outer boundary and the hypersurface in the Schwarzschild manifold. In 3-dimension, this inequality is equivalent to the Riemannian Penrose inequality. This talk is based on joint work with Siyuan Lu.