Persistence of first integrals under integrable deformations for certain holomorphic foliation singularities

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Resumo/Abstract:
We consider analytic deformations of holomorphic foliations. In the first part we study deformations that embed into a holomorphic foliation. We address the local framework, in a neighborhood of the origin $0 \in \mathbb{C}^{n+1}$, which is supposed to be a singular point. The starting foliation has a holomorphic first integral, i.e., it is of the form $df = 0$ for a germ of holomorphic function $f \in \mathcal{O}_n$ at the origin $0 \in \mathbb{C}^n, n \geq 3$. We also assume that the germ f is irreducible and reduced. Our central hypotheses is that, *outside of a dimension* $\leq n-3$ *analytic subset* $Y \subset X$*, the analytic hypersurface* $X_f : (f = 0)$ *has only normal crossings singularities.* We then prove that, as a germ, the developing foliation also exhibits a holomorphic first integral. As an application of our techniques we are able to the persistence of a first integral for perturbations of a form $\omega = df + f\eta$ where f is germ as above and quasi-homogeneous.

In the second part of the paper, we consider analytic deformations $\{_t\}_{t\in\mathbb{C},0}$, of a local pencil $_0 : f^n/g^m = const.$, for $f, g \in \mathcal{O}_n$ in general position. Under some generic geometric conditions on f and g we conclude the existence of a meromorphic first integral for the foliation in dimension $n+1$.