Parafermionic observables in 2D statistical physics

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Resumo/Abstract:
In the early eighties, physicists Belavin, Polyakov and Zamolodchikov postulated conformal invariance of critical planar statistical models. This prediction enabled physicists to harness Conformal Field Theory in order to formulate many conjectures on these models. From a mathematical perspective, proving rigorously the conformal invariance of a model (and properties following from it) constitutes a formidable challenge. In recent years, the connection between discrete holomorphicity and planar statistical physics led to spectacular progress in this direction. Kenyon, Chelkak and Smirnov exhibited discrete holomorphic observables in the dimer and Ising models and proved their convergence to conformal maps in the scaling limit. These results paved the way to the rigorous proof of conformal invariance for these two models.

Other discrete observables have been proposed for a number of critical models, including self-avoiding walks and Potts models. While these observables are not exactly discrete holomorphic, their discrete contour integrals vanish, a property shared by discrete holomorphic functions. This property sheds a new light on the critical models, and we propose to discuss some of its applications.

We will sketch the proof of a conjecture made by Nienhuis regarding the connective constant of the hexagonal lattice. More precisely, we will compute (joint work with Smirnov) the rate of growth of the number of self-avoiding walks of length n starting at the origin.

We will also discuss the absence of spontaneous magnetization for the critical planar Potts models with 2, 3 or 4 colors (joint work with Sidoravicius and Tassion), a fact conjectured by Baxter. Both results are based on observables, called parafermionic observables, possessing the property mentioned above. We will conclude the talk by explaining why establishing a slightly stronger property of these observables would lead to a proof of conformal invariance for these models.