Workshop em Fundamentos da Ciência da Computação: Algoritmos Combinatórios e Estruturas Discretas

IMPA, Rio de Janeiro, 10–13 de abril de 2006

Programme

	Mo (10/4)	**Tu (11/4)**	**We (12/4)**	**Th (13/4)**
9:30–10:15	Füredi	Duffus	Kostochka	Mubayi
10:15–10:45	**Coffee break**			
10:45–11:30	Moreira	Szwarcfiter	Skokan	Lemos
11:30–14:00	**Lunch break**			
14:00–14:45	Fernandes	Marciniszyn	Lopes	
14:45–15:30	Chataigner	Dellamonica		

- Füredi: *Sets of few distances in high-dimensional normed spaces*
- Moreira: *Measuring sets of infinite sequences with bounded complexity*
- Fernandes: *Approximation results on rational objectives*
- Chataigner: *On balanced connected partitions of graphs*
- Duffus: *Automorphisms and endomorphisms of finite partially ordered sets*
- Szwarcfiter: *Linear time representation and recognition of unit circular-arc graphs*
- Marciniszyn: *Two results from extremal graph theory*
- Dellamonica: *Yet another result from extremal graph theory*
- Kostochka: *Domination in cubic connected graphs*
- Skokan: *Ramsey-type questions for graphs and hypergraphs*
- Lopes: *Graded forests and rational knots*
- Mubayi: *Stability in extremal set theory*
- Lemos: *Weight distribution of the bases of a graph (or a matroid)*

Abstracts

Sets of few distances in high-dimensional normed spaces
Zoltán Füredi, U. of Illinois at Urbana-Champaign and Rényi Institute

This talk is about isosceles triangles.

It is known that an equidistant set in the d-dimensional Euclidean space E^d can have at most $d+1$ elements. Let $f(\mathbf{N}, k)$ denote the maximum size of a set in the normed space $\mathbf{N}$ with at most k pairwise distinct distances. Alon and Pudlák (2003) proved that $f(\ell_p^d, 1) = O(d \log d)$ for any fixed odd integer p. For general normed spaces the upper bound is $f(\mathbf{N}, k) \leq 2^{kd}$ (Petty 1971 for $k = 1$ and Swaenapol 1999 for all k). The point-set $\{0, 1, \ldots, k\}^n$ gives $f(\ell_\infty^d, k) \geq (k+1)^d$. Here we consider further generalizations:

A set $P \subset \mathbb{R}^d$ in the normed space $\mathbf{N} = (\mathbb{R}^d, \| \ \|)$ is called k-dependent if $\forall X \subset P$,

$$|X| = k \text{ determines less than } \tbinom{k}{2} \text{ distances.}$$

For example, for $k = 3$, every triangle is isosceles. Let $g(\mathbf{N}^d, k)$ denote the maximum size of such a set. The point set $\{1, \ldots, \binom{k}{2}\}^d$ shows

$$g(\ell_\infty^d, k) \geq \binom{k}{2}^d.$$

Our aim is to prove that there exists c dependent only on d, k such that $g(\mathbf{N}^d, k) \leq c$, independently on the norm. We show,

$$g(\mathbf{N}^d, k) \leq \left(\frac{1}{2}(k-1)(k^2 - 2k + 6)\right)^{c3^d d \log d}.$$

Measuring sets of infinite sequences with bounded complexity
Carlos Gustavo Moreira, IMPA

Given an infinite sequence over a finite alphabet with b letters (which can be identified with a real number written in base b), its complexity function is a function p defined on the positive integers such that $p(n)$ is the number of factors (subsequences of consecutive terms) of length n of our infinite sequence. Given a function $f(n)$, we consider the set of all infinite sequences

(or real numbers) with complexity bounded by f. We give lower and upper estimates for the number of sequences of size n which appear as factors of some sequence in this set. When we consider sets of real numbers with complexity bounded by f, it is not difficult to show that they have positive Hausdorff dimension if and only if f grows exponentially fast. We study this sets in general (which, in the more interesting cases, have zero Hausdorff dimension), and estimate generalized fractal dimensions of them, associated to more general gauge functions than the functions x^s, $s > 0$ (which are used to define the Hausdorff dimension).

This is joint work with Christian Mauduit (IML, Marseille).

Approximation results on rational objectives
Cristina Gomes Fernandes, USP

We address the problem of finding approximate solutions for a class of combinatorial optimization problems with rational objectives. We show that, if there is an α-approximation for the problem of minimizing a nonnegative linear function subject to constraints satisfying a certain *increasing* property then there is an α-approximation ($1/\alpha$-approximation) for the problem of minimizing (maximizing) a nonnegative rational function subject to the same constraints. Our framework applies to covering integer programming problems and network design problems, among others.

This is joint work with José R. Correa (Adolfo Ibañez, Santiago) and Yoshiko Wakabayashi (USP).

On balanced connected partitions of graphs
Frédéric Chataigner, USP

Let (G, w) denote a pair consisting of a connected graph G and a weight function $w : V(G) \to \mathbb{Z}_+$. For $X \subseteq V(G)$, let $w(X)$ denote the sum of the weights of the vertices in X. We consider the following problem: given a pair (G, w) and a positive integer $q \geq 2$, find a q-partition $P = (V_1, V_2, \ldots, V_q)$ of $V(G)$ such that $G[V_i]$ is connected $(1 \leq i \leq q)$ and P maximizes $\min\{w(V_i) : 1 \leq i \leq q\}$. This problem is called *Max Balanced Connected Partition* and is denoted by BCP. When $q \geq 2$ is fixed, we denote the problem by BCP_q. We show some approximability and inapproximability results for both BCP and BCP_2. These results include a proof that BCP does not admit a PTAS unless P = NP and a 2-approximation algorithm for BCP_3 on 3-connected graphs. We also mention some other known algorithmic results.

This is joint work with Liliane Salgado (UFPE) and Yoshiko Wakabayashi (USP).

Automorphisms and endomorphisms of finite partially ordered sets
Dwight Duffus, Emory University

It has been known for some time that a partially ordered set of order n must have at least 2^n endomorphisms, or order preserving maps into itself. Moreover, most ordered sets have no nontrivial automorphisms. However, it is still not known if the

$$c(n) = \min\left\{\frac{|\mathrm{End}(P)|}{|\mathrm{Aut}(P)|} : \text{with the minimum over all } P \text{ of order } n\right\},$$

tends to infinity with n, in complete generality, though it has been demonstrated in many situations. We examine results for the case of bipartite orders, obtained jointly with Tomasz Łuczak.

Linear time representation and recognition of unit circular-arc graphs
Jayme L. Szwarcfiter, UFRJ

In a recent paper, Durán, Gravano, McConnell, Spinrad and Tucker described an algorithm of complexity $O(n^2)$ for recognizing whether a graph G with n vertices and m edges is a unit circular-arc (UCA) graph. Furthermore the following open questions were posed in the above paper:

(i) Is it possible to construct a UCA model for G in polynomial time?

(ii) Is it possible to construct a UCA model, whose extremes of the arcs correspond to integers of polynomial size?

(iii) If (ii) is true, could such a model be constructed in polynomial time?

In the present paper, we describe a characterization of UCA graphs which leads to a different recognition algorithm and to answering these questions, in the affirmative. We construct a UCA model whose extremes of the arcs correspond to integers of size $O(n)$. The proposed algorithms, for recognizing UCA graphs and constructing UCA models, have complexities $O(n + m)$. Furthermore, the complexities reduce to $O(n)$, if a proper circular-arc (PCA)

model of G is already given as the input, provided the extremes of the arcs are ordered. We remark that a PCA model of G can be constructed in $O(n+m)$ time, using the algorithm by Deng, Hell and Huang.

This is joint work with Min Chih Lin (Universidad de Buenos Aires).

Two results from extremal graph theory
Martin Marciniszyn, ETH Zürich

In the first result, we prove the existence of many complete graphs in almost all sufficiently dense partitions obtained by an application of Szemerédi's regularity lemma. More precisely, we consider the number of complete graphs K_l on l vertices in l-partite graphs where each partition class consists of n vertices and there is an ϵ-regular graph on m edges between any two partition classes. We show that for all $\beta > 0$, at most a β^m-fraction of all such graphs contain less than the expected number of copies of K_l provided ϵ is sufficiently small, $m \geq n^{2-1/(\ell-1)}$, and n is sufficiently large. This result is a counting version of a restricted version of a conjecture by Kohayakawa, Łuczak and Rödl from 1997, and it is well known that this result implies several results for random graphs. This is joint work with S. Gerke and A. Steger.

In the second result, we prove a probabilistic version of the famous theorem by Erdős and Gallai from 1959 that guarantees the existence of long cycles in graphs with sufficiently many edges. For any constant $\alpha > 0$, suppose that an adversary deletes $(\alpha - o(1))pn^2/2$ edges from a random graph $G(n,p)$. Then asymptotically almost surely there remains a cycle of length $(1-\alpha)n$ in the graph provided that $p \gg n^{-1}$. This is joint work with D. Dellamonica Jr, Y. Kohayakawa, and A. Steger.

Yet another result from extremal graph theory
Domingos Dellamonica Jr, USP

The following game is proposed. Let $G = G_{n,p}$ be a random graph on n vertices with edge probability $p \gg n^{-1/2}$. An adversary maliciously removes up to a little less than half the edges incident to every vertex of G, thus obtaining a graph G'. We prove that asymptotically almost surely G' contains a Hamiltonian cycle.

This is joint work with Y. Kohayakawa, M. Marciniszyn, and A. Steger.

Domination in cubic connected graphs
Alexandr V. Kostochka, U. of Illinois at Urbana-Champaign

In 1996, Reed proved that the domination number $\gamma(G)$ of every n-vertex graph G with minimum degree at least 3 is at most $3n/8$ and conjectured that if G is cubic and connected, then $\gamma(G) \leq \lceil n/3 \rceil$. Kawarabayashi, Plummer and Saito proved that the conjecture is 'close to the truth' for cubic graphs of large girth. We disprove the conjecture by constructing a sequence $\{G_k\}_{k=1}^{\infty}$ of connected cubic graphs with $\lim_{k\to\infty} \frac{\gamma(G_k)}{|V(G_k)|} \geq \frac{8}{23} = \frac{1}{3} + \frac{1}{69}$. On the other hand, we show that $\gamma(G) \leq 4n/11$ for every n-vertex cubic connected graph G if $n > 8$. Some more facts on the domination number of cubic graphs will be discussed. The talk is based on joint work with B. Stodolsky.

Ramsey-type questions for graphs and hypergraphs
Jozef Skokan, USP

Graded forests and rational knots
Pedro Lopes, Instituto Superior Técnico e IMPA

Rational knots are important in many respects, in particular for their connections with DNA.

We recently produced a new algorithm for retrieving the determinant of any rational knot which is based on graphs. More specifically, we associate a graded forest to any rational knot. This allows us to write down its determinant in terms of the numbers of half-twists in each tassle of the rational knot.

In this talk we elaborate on these graded forests and rational knots. This is joint work with Louis H. Kauffman.

References

1. L. H. Kauffman, S. Lambropoulou, On the classification of rational tangles, *Adv. in Appl. Math.*, 33 (2004), no. 2, 199–237
2. L. H. Kauffman, P. Lopes, Color spectra and coloring conjectures, `arXiv:math.GT/0512088`, submitted

Stability in extremal set theory
Dhruv Mubayi, University of Illinois at Chicago

I will begin by defining the notion of stability for monotone properties of set systems. This formulation encompasses the classical definition in extremal graph theory initiated by Erdős and Simonovits in the 60s. Various stability theorems about classical intersection-type questions will be given, for example, I will show that a nontrivial intersecting family of k-sets of almost maximum size has structure close to that guaranteed by the extremal examples of the Hilton–Milner theorem. Finally, I will indicate how the stability approach can be used to prove an exact result in extremal set theory.

Weight distribution of the bases of a graph (or a matroid)
Manoel Lemos, UFPE

Given a weighted graph, let $w_1, w_2, \ldots, w_n$ denote the increasing sequence of all possible distinct spanning tree weights. In 1992, Mayr and Plaxton proved the following conjecture proposed by Kano: every spanning tree of weight w_1 is at most $k-1$ edge swaps away from some spanning tree of weight w_k. We will talk about the extension of this result for matroids. We will also prove that all the four conjectures due to Kano hold for matroids provided one partitions the bases of a matroid by the weight distribuition of its elements instead of their weight.
