Sign Regularity of Maclaurin Coefficients of Entire Functions in the Laguerre-Pólya Class

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Resumo/Abstract:
The Laguerre-Pólya class, denoted by $\mathcal{LP}$, is the completion of real polynomials with only real zeros in the topology of the locally an uniformly convergence. If $\phi(z) = \sum a_n z^n$ is the Maclaurin expansion of an entire function from $\mathcal{LP}$ of order two we investigate the distribution of signs in the sequence (a_n) and observe a rather regular pattern in the sense that the block of signs $++--$ occurs very frequently. We prove that, roughly speaking, if ϕ is of order two, possesses only real zeros and the pattern $a_{n-1}a_{n+1} \leq 0$ holds asymptotically then $\phi \in \mathcal{LP}$. Nevertheless, for any ρ with $0 \leq \rho \leq 2$, we provide examples of entire functions of order ρ belonging to the Laguerre-Pólya class such that the pattern $a_{n-1}a_{n+1} \leq 0$ fails asymptotically.