

K-functionals of fractional order: characterizations and applications

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Resumo/Abstract:

As a tool to express intrinsic properties, closely related to the smoothness, and some approximation properties of a function K-functionals play an important role in approximation theory. In most cases their characterizations via rate of approximation of average operators are desired.

On the euclidean space and on the sphere we have an important characterization given by the rate of approximation of spherical mean operators (Dai & Ditzian (2004)). Application of it on the spherical setting, namely, in the study of decay rates of eigenvalues sequence of certain integral operators has shown the importance of such characterizations in order to improve results already known (Kühn (1987)). Details can be found in Jordão, Menegatto & Sun (2014) and Castro & Jordão (2017).

Further it is designed two types of a general type of spherical mean operators, depending on a real number as parameter, and they are employed to approximate L_p class functions. One of them is very useful theoretically, it generalizes a class of combinations of average operators given in Dai & Ditzian (2004). The second one brings us a more useful configuration in the sense of applications, and by applications here we mean computacional calculations.

It is shown that optimal orders of approximation of these operators are achieved via appropriately defined K-functionals of fractional orders. Asymptotic relations between the rate of approximation of the new operators and the K-functional of fractional order are established. When the parameter we work with is taken as a natural number the general type of spherical mean operator, the K-functional and also the result relating such objects turn out the same as in Dai & Ditzian (2004) which introduces a class of “multi-layered” spherical mean operators. More details can be found in Jordão & Sun (2015) and Castro & Jordão (2017).

References

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