Local Hardy-Sobolev inequalities for canceling elliptic differential operators

Tiago H. Picon (FFCLRP-USP) picon@ffclrp.usp.br, Jorge Hounie (UFSCar)

Resumo/Abstract:
In this lecture we show that if $A(x, D)$ is a linear differential operator of order ν with smooth complex coefficients in $\Omega \subset \mathbb{R}^N$ from a complex vector space E to a complex vector space F, then the Hardy-Sobolev inequality

$$\int_{\mathbb{R}^N} \frac{|D^{\nu-\ell}u(x)|}{|x-x_0|^\ell}\, dx \leq C \int_{\mathbb{R}^N} |A(x,D)u| dx, \quad u \in C_c^\infty(B; E),$$

for $\ell \in \{1, ..., \min\{\nu, N-1\}\}$ holds locally at any point $x_0 \in \Omega$ if and only if $A(x, D)$ is elliptic and the constant coefficients homogeneous operator $A_\nu(x_0, D)$ is canceling in the sense of Van Schaftingen for every $x_0 \in \Omega$ which means that

$$\bigcap_{\xi \in \mathbb{R}^N \setminus \{0\}} a_\nu(x_0, \xi)[E] = \{0\}.$$

Here $A_\nu(x, D)$ is the homogeneous part of order ν of $A(x, D)$ and $a_\nu(x, \xi)$ is the principal symbol of $A(x, D)$.