

3D domino tilings

Nicolau C. Saldanha (PUC-Rio), J. Freire, T. Kittipassorn, C. Klivans e P. Milet

Resumo/Abstract:

Domino tilings in two dimensions have been extensively studied; 3D tilings are not nearly so well understood. In this talk we present some results and conjectures on 3D domino tilings. Our emphasis is on local moves, particularly the flip (removing two dominoes and placing them in a different position).

In two dimensions, the set of tilings of a simply connected subset of the plane is connected and simply connected via flips; if the region is not simply connected or not planar there is an invariant (the flux) and connected components are not always simply connected.

In three dimensions the obvious generalization does not hold: for instance, the set of tilings of the $4 \times 4 \times 4$ box has 93 connected components. We prove more generally that for boxes there is an integer valued invariant, the twist. In more general regions, the twist may assume values in \mathbb{Z} or in a quotient $\mathbb{Z}/(m)$ (here the modulus m is a function of the flux). Flux and twist together are not complete invariants, not even in boxes; the $4 \times 4 \times 4$ box has many such counterexamples. Flux and twist are, however, complete invariants if we allow for refinements. Empirical evidence also indicate that they are almost complete in a probabilistic sense.