Khintchine's logarithmic law for continued fractions with sequentially restricted entries

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Non-stationary shift spaces are models of sequential dynamical system who were intensively studied in order to construct symbolic models for ergodic automorphism (Vershik) or in the context of the isomorphism problem of shift spaces (Krieger). Recently, the focus moved towards thermodynamical formalism and related questions.

In the talk, a possible adaption of Ruelle's operator theorem to this setting is presented. That is, in order to circumvent the intrinsic non-existence of eigenfunctions, the theorem establishes geometric ergodicity for a family of ratios of operators. As a corollary, one obtains uniqueness of measures and non-uniqueness of eigenfunctions. These results have applications to a classical problem in metric number theory. For a sequence (α_n) converging to ∞, set

$$X_\alpha := \left\{ x = \frac{1}{x_1 + \frac{1}{x_2 + \cdots}} : x_n \in \mathbb{N}, x_n \geq \alpha_n \text{ for all } n \right\}.$$

That is, X is the subset of $[0, 1]$ such that the n-th entry of the continued fraction expansion of each element is bigger than or equal to α_n. In this setting, it is possible to deduce a law of the iterated logarithm for square integrable functions from geometric ergodicity (joint work with Xuan Zhang). If, e.g., $\alpha_n := n^n$, one obtains that X_α has Hausdorff dimension 1/2 and that

$$\limsup_{n\to\infty} \left(\frac{1}{n^n} \prod_{k=1}^{n} \sqrt[k]{x_k} \right)^{\frac{1}{\sqrt{n \log \log n}}} = \sqrt{2} \text{ a.s.}$$