A Weight Invariant Space Approach to Proving Estimates for Singular Integral Operators

Jarod V. Hart (Kansas University - KU) jvhart@ku.edu, Lucas Oliveira, Rodolfo H. Torres

Resumo/Abstract:
In this talk, we discuss some weight invariant type properties for Sobolev-BMO spaces, and use these properties to prove weighted Hardy space estimates for Calderón-Zygmund operators. Roughly speaking, Sobolev-BMO spaces $I_s(BMO)$ for $s \geq 0$ are made up of s order anti-derivatives of BMO functions, where BMO is the John-Nirenberg space of Bounded Mean Oscillation. The weighted counterparts of these spaces $I_s(BMO_w)$ are defined by replacing the Lebesgue measure dx in the Sobolev-BMO norms with a Muckenhoupt weighted measure $w(x)dx$. These spaces are very resilient to weight perturbations, which we show by proving $I_s(BMO) = I_s(BMO_w)$ for any Muckenhoupt weight $w \in A_\infty$. We use this weight invariant result to prove that a Calderón-Zygmund operator T, satisfying appropriate cancellation conditions, is bounded on the weighted Hardy space H^p_w when $p_0 < p < \infty$ and $w \in A_{p/p_0}$; here $0 < p_0 < 1$ depends on the operator T. Interestingly, these estimates do not collapse to the known Lebesgue space theory for Calderón-Zygmund operators when $1 < p < \infty$. We will also discuss some deep connections between weight invariance properties and power-type John-Nirenberg inequalities for BMO spaces.