Orthogonal Polynomials and Sharp Strichartz Estimates

Felipe Gonçalves (University of Alberta) felipe.goncalves@ualberta.ca

Resumo/Abstract:
In 1977 Strichartz showed that if $\frac{d}{p} + \frac{2}{q} = \frac{d}{2}$ then

$$\| \| e^{i\Delta t} f(x) \|_{L^p_x} \|_{L^q_t} \leq C_{p,d} \| f(x) \|_{L^2_x}$$

for any $f \in L^2_x$, where d denotes the dimension and $e^{i\Delta t}$ is the Schrödinger time-evolution operator. Since then, it is conjectured that Gaussian functions are the only maximizer of the above inequality. This conjecture is known to be true for only three particular cases of exponents: $(p, q, d) \in \{(6, 6, 1); (4, 8, 1); (4, 4, 2)\}$. This exponents all share the crucial property of p being an even integer and p/q being an integer. This fact changes the problem to a $(L^2 \mapsto L^2)$–type estimate with flavors of a Fourier restriction problem. In this talk we present a new approach for these sharp estimates via Hermite and Laguerre polynomial expansions.