Positive measures on the unit circle from specialized pairs of real sequences

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Resumo/Abstract:
Given a sequence of complex numbers $\{\alpha_n\}_{n\geq 0}$, where $|\alpha_n| < 1$, $n \geq 0$, it is known that associated with this sequence there exists a unique positive measure μ on the unit circle such that the sequence of monic polynomials $\{\Phi_n\}_{n\geq 0}$ generated by

$$\Phi_{n+1}(z) = z\Phi_n(z) - \overline{\alpha}_n \Phi_n^*(z), \quad n \geq 0,$$

are the sequence of monic orthogonal polynomials on the unit circle with respect to μ. This result known in the past as Favard theorem on the unit circle has come to be known in recent years as the Verblunsky theorem. The elements α_n are also now referred to as Verblunsky coefficients associated with the measure μ. There has been considerable amount of work on the topic of looking for restrictions in the coefficients α_n that will guarantee particular properties satisfid by the measure μ. In this work we look into some recent developments in this topic of research, where the starting point is not $\{\alpha_n\}_{n\geq 0}$ but specialized pairs of real sequences $[\{c_n\}_{n\geq 1}, \{d_{n+1}\}_{n\geq 1}]$ which can be easily derived from $\{\alpha_n\}_{n\geq 0}$.