Bounding $S_n(t)$ on the Riemann hypothesis

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Resumo/Abstract:
Let $S(t)$ denote the argument of the Riemann zeta-function at the point $1/2 + it$. Let $S_n(t)$ be the n-th antiderivative of $S(t)$ (adding a suitable constant c_n at each step). In 1924, J. Littlewood established, under the Riemann hypothesis, that

$$S_n(t) \ll \frac{\log t}{(\log \log t)^{n+1}},$$

and this estimate has never been improved in its order of magnitude over the last 92 years. The efforts have focused in improving the implicit constant in this estimate. In this talk we will show how to obtain the best (up to date) form of all of these estimates. This involves the use of certain special entire functions of exponential type. This is an aplication of approximation theory in analytic number theory.