Ekeland's variational principle and local surjection theorems

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Resumo/Abstract:
If $\mathrm{F}(0) = 0$, it is a very old idea to solve $\mathrm{F}(\mathrm{x}) = \mathrm{y}$ near 0 by assuming that $\mathrm{DF}(0)$ is invertible. This is the content of the inverse function theorem, which is proved by constructing a sequence of approximate solutions which converge to the true solution. In practice, it is disappointing, because the size of the neighbourhood of 0 where the function can be inverted is extremely small. I this talk, I will show that, if one does not ask for uniqueness, that is, if one does not require the solution to be unique, then the size of that neighbourhood changes by orders of magnitude. Iterations no longer converge, and a new (and simpler) proof is required, relying on Ekeland's variational principle. The proof will be given, and several extensions and applications. We will give conditions under which there is a continuous section, i.e. a continuous function $\mathrm{G}(\mathrm{y})$ such that $\mathrm{FoG}(\mathrm{y}) = \mathrm{y}$, we will give an application to a singular perturbation proble, and we will prove a Morse Lemma under very weak assumptions.