On subtowers of towers of function fields

María Chara[1], Horacio Navarro[2], Ricardo Toledano[3]

[1] Instituto de Matemática Aplicada del Litoral (UNL-CONICET), Santa Fe, Argentina

[2] Universidad del Valle, Cali, Colombia

[3] Universidad Nacional del Litoral, Santa Fe, Argentina

Let F be an algebraic function field of one variable over the finite field Fq. In 1881, Ihara introduced the function A(q) which measures how large can be the number of rational places in function fields with respect to their genus. It is not much known about this quantity but it importance relies in the fact that positive lower bounds for this function imply the existence of arbitrary long codes with good parameters. The first examples of general lower bounds for Ihara's function involved deep results from class field theory and modular curves. The problem with these kind of constructions is that they do not provide explicit representation of the involved function fields, which are needed for the explicit construction of asymptotically good codes.

Another way of obtaining non-trivial lower bound for Ihara's function is through the construction of asymptotically good towers of function fields. The first breakthrough in this setting came from the hands of Garcia and Stichtenoth who exhibited explicit towers of function fields with asymptotically good limits and using only basic results on ramification in separable extensions of function fields. In many of these towers, all the steps are simultaneously defined by the same equation. Towers defined in this way are called recursive.

In this talk we will provide a method to whether construct subtowers of function fields from already studied towers or to check if two apparently different equations define towers which are subtowers one of the other. We will prove that the given method actually give rise to proper subsequences of a given tower. An interesting feature of these results is that they can be easily implemented in a computer so we were able to search for many equations defining subtowers.